ESERCIZIO 1. 13/02/2012

① $\mathcal{H} = \sum_{i=1}^{N} h^{(1)}(i)$

$$h^{(1)} = \frac{p^2}{2m} + U \ln\left[\left(\frac{r}{R}\right)^{\alpha}\right] \qquad 0<r<R$$

$$Q_N(V,T) = \frac{1}{N!} q^N$$

$$q = \frac{1}{h^3}\int d\bar{r} \int d\bar{p}\, e^{-\beta h} = \frac{1}{h^3}\int d\bar{p}\, e^{-\frac{\beta p^2}{2m}} \int_{0<r<R} d\bar{r}\, e^{-\beta U \ln\left(\frac{r}{R}\right)^{\alpha}}$$

$$= \frac{1}{\lambda^3} 4\pi \int_0^R dr\, r^2\, e^{-\ln\left(\frac{r}{R}\right)^{\beta U \alpha}}$$

$$\beta U \alpha \equiv \gamma < 3$$

$$= \frac{4\pi}{\lambda^3}\int_0^R dr\, r^2 \left(\frac{R}{r}\right)^{\gamma} = \frac{4\pi}{\lambda^3} R^{\gamma} \int_0^R dr\, r^{2-\gamma}$$

$$= \frac{4\pi}{\lambda^3}\frac{R^{\gamma}}{3-\gamma} R^{3-\gamma} = \frac{1}{\lambda^3}\frac{4\pi}{3-\gamma} R^3$$

$$\boxed{Q_N(V,T) = \frac{1}{N!}\left[\frac{4\pi}{3-\gamma}\frac{R^3}{\lambda^3}\right]^N \simeq \left[\frac{e}{N}\frac{4\pi}{3-\gamma}\frac{R^3}{\lambda^3}\right]^N}$$

$$\boxed{A(N,V,T) = -N K_B T \ln\left[\frac{e}{N}\frac{V}{\lambda^3}\frac{3}{3-\gamma}\right] \quad V = \frac{4\pi}{3}R^3}$$

$$A(N,V,T) = -N K_B T \ln\left[\frac{eV}{N\lambda^3}\frac{3}{3-\beta U \alpha}\right]$$

(2) $$E = \frac{\partial \beta A}{\partial \beta} = -N \frac{\partial}{\partial \beta} \ln\left[\frac{eV}{N\lambda^3} \frac{3}{3-\beta U\alpha}\right]$$

$$= -N \frac{\partial}{\partial \beta}\left[\ln[\beta^{-3/2}] - \ln[3-\beta U\alpha]\right]$$

$$\boxed{E = N k_B T\left[\frac{3}{2} - \frac{\beta U\alpha}{3-\beta U\alpha}\right]}$$ $\lambda \sim \beta^{1/2}$

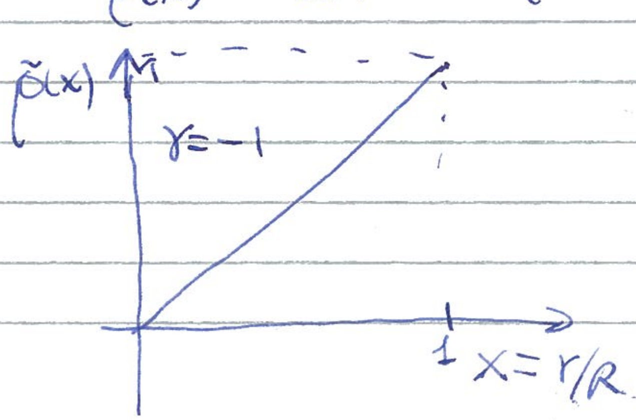
(3) $$S = \frac{E-A}{T} = N k_B\left[\frac{3}{2} - \frac{\beta U\alpha}{3-\beta U\alpha} + \ln\left(\frac{eV}{\lambda^3 N}\frac{3}{3-\beta U\alpha}\right)\right]$$

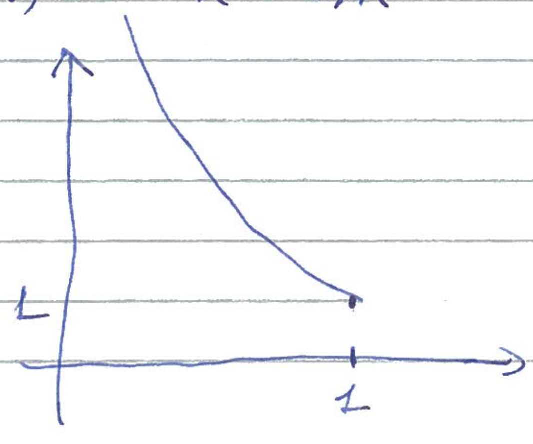
(4) $$\rho(r) = N\langle\delta(\bar r-\bar r_1)\rangle = N\frac{\int d\bar r_1\, \delta(\bar r-\bar r_1)\, e^{-\beta U(\bar r_1)}}{\int d\bar r_1\, e^{-\beta U(\bar r_1)}}$$

$$= \frac{N e^{-\beta U(r)}}{\frac{4\pi}{3-\gamma}R^3} = N\frac{(R/r)^\gamma}{\frac{4\pi}{3-\gamma}R^3} = \frac{N}{\frac{4\pi R^3}{3}}\frac{3-\gamma}{3}\left(\frac{R}{r}\right)^\gamma$$

$$\rho_0 = \frac{N}{V} \Rightarrow \boxed{\rho(r) = \rho_0\frac{3-\gamma}{3}\left(\frac{R}{r}\right)^\gamma}$$

$$\frac{\rho(r)}{\rho(R)} = \left(\frac{R}{r}\right)^\gamma \equiv \tilde\rho\left(\frac{R}{r}\right) \equiv \tilde\rho(x) \qquad x = r/R$$





ESERCIZIO 2. 13/02/2012

①

$$-\frac{\partial}{\partial\beta}\ln \mathcal{Z}\Big|_{z,V} = \frac{1}{\mathcal{Z}}\sum_{N=0}^{\infty} z^N \sum_\alpha E_\alpha e^{-\beta E_\alpha}$$

$$= \sum_{N=0}^{\infty} \frac{z^N Q_N}{\mathcal{Z}} \frac{1}{Q_N}\sum_\alpha E_\alpha e^{-\beta E_\alpha}$$

$$= \sum_{N=0}^{\infty} \frac{z^N Q_N}{\mathcal{Z}} \langle E\rangle_{CAN} = \langle E\rangle_{G.CAN} = E(z,V,\beta)$$

②

$$-\frac{\partial}{\partial\beta}\left[-\frac{\partial}{\partial\beta}\ln \mathcal{Z}\right]\Big|_{z,V} = \frac{1}{\mathcal{Z}}\sum_{N=0}^{\infty} z^N \sum_\alpha E_\alpha^2 e^{-\beta E_\alpha}$$

$$+ \frac{1}{\mathcal{Z}^2}\frac{\partial \mathcal{Z}}{\partial\beta}\sum_{N=0}^{\infty} z^N \sum_\alpha E_\alpha e^{-\beta E_\alpha}$$

$$= \langle E^2\rangle_{G.CAN} - \left(\langle E\rangle_{G.CAN}\right)^2 \equiv \langle(\Delta E)^2\rangle_{G.CAN.}$$

③ Gas di Bosoni

$$\ln \mathcal{Z} = \frac{V g_{5/2}(z)}{\lambda^3} - \ln(1-z) \qquad \lambda^3 \sim \beta^{3/2}$$

$$\boxed{E(z,V,\beta) = -\frac{\partial \ln \mathcal{Z}}{\partial\beta}\Big|_{z,V} = \frac{3}{2}\frac{V g_{5/2}(z)}{\beta\lambda^3}}$$

④

$$\langle\Delta E^2\rangle_{G.CAN} = -\frac{\partial}{\partial\beta}\left[\frac{3}{2}\frac{V g_{5/2}(z)}{\beta\lambda^3}\right]\Bigg|_{z,V} = \frac{5}{2}\frac{3}{2}\frac{V g_{5/2}(z)}{\beta^2\lambda^3}$$

$$\boxed{\frac{\sqrt{\langle(\Delta E)^2\rangle}}{\langle E\rangle^2}} = \left[\frac{15}{4}\frac{V g_{5/2}(z)}{\beta^2\lambda^3}\,\frac{4}{9}\frac{\beta^2(\lambda^3)^2}{V^2 g_{5/2}^2(z)}\right]^{\frac{1}{2}} = \left[\frac{5}{3}\frac{1}{g_{5/2}(z)}\right]^{\frac{1}{2}}\left(\frac{\lambda^3}{V}\right)^{\frac{1}{2}}$$