Exercise 1

① For a linear chain

$$\omega(q) = 2\sqrt{\frac{G}{M}}\left|\sin\left(\frac{qa}{2}\right)\right| \simeq \cancel{2}\sqrt{\frac{G}{M}}\frac{a}{\cancel{2}}|q|, q\to 0$$

and for small $|q|$

$$\boxed{\omega(q) = c|q| \quad , \quad c = \sqrt{\frac{G}{M}}a}$$

② In 1-D $\epsilon = 1$ and

$$u(n,t) = \sum_q \frac{\ell a}{5L}\frac{1}{1+(q\ell)^2}\cdot 2\cdot\cos[qna - c|q|t]$$

$$= \frac{\cancel{L}}{\cancel{2}\pi}\frac{\ell a}{5\cancel{L}}\cdot\cancel{2}\int_{-\infty}^{\infty}dq\,\frac{\cos[qna - c|q|t]}{1+(q\ell)^2}$$

$$= \frac{\ell a}{5\pi}\left\{\int_0^\infty dq\,\frac{\cos[qna - cqt]}{1+(q\ell)^2} + \int_{-\infty}^0 dq\,\frac{\cos[qna + cqt]}{1+(q\ell)^2}\right\}$$

$$= \frac{\ell a}{5\pi}\int_0^\infty dq\left\{\frac{\cos[q(na - ct)]}{1+(q\ell)^2} + \frac{\cos[q(na+ct)]}{1+(q\ell)^2}\right\}$$

$$u(n,t) = \frac{\cancel{\ell}a}{5\cancel{\pi}}\frac{\cancel{\pi}}{2\cancel{\ell}}\left[e^{-|na-ct|/\ell} + e^{-|na+ct|/\ell}\right]$$

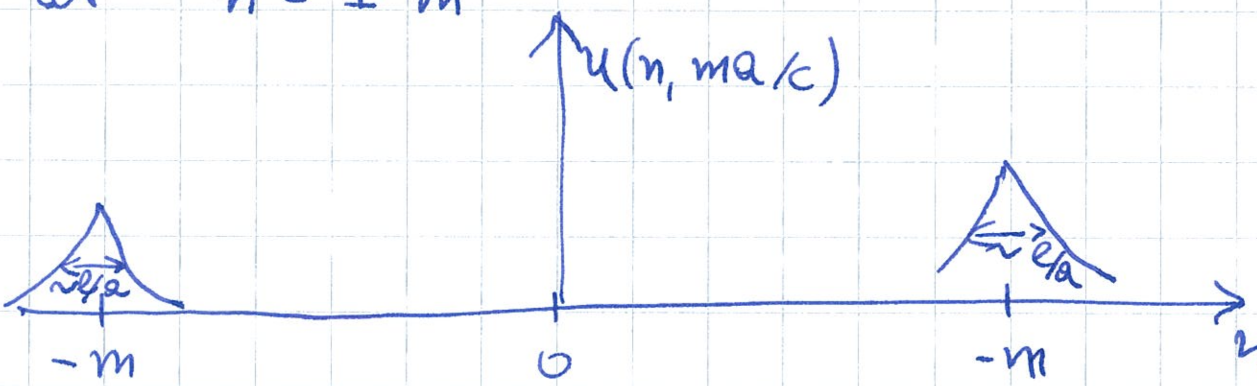
$$\boxed{u(n,t) = (a/10)\left[e^{-|na-ct|/\ell} + e^{-|na+ct|/\ell}\right]}$$

③ Those near the origin within a window of width $\sim \ell$

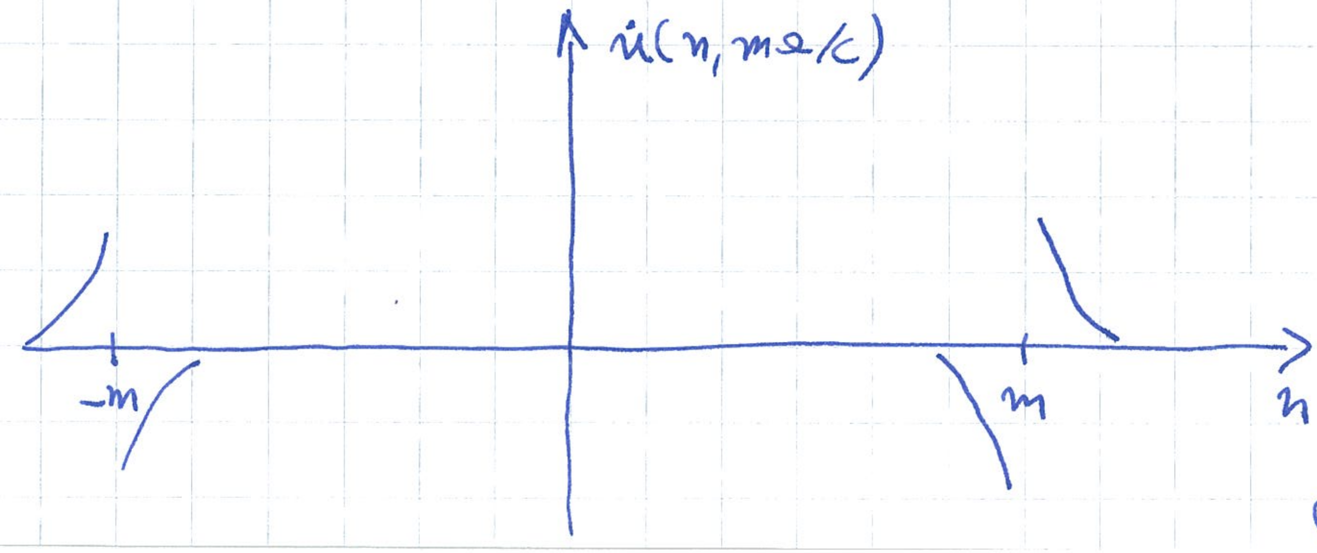
④ $$\boxed{\dot{u}(n,t=0) = \frac{a}{10\ell} c \left[\frac{n}{|n|} e^{-|n|a/\ell} - \frac{n}{|n|} e^{-|n|a/\ell} \right] = 0}$$

⑤ $$u(n, ma/c) = \frac{a}{10} \left[e^{-|n-m|a/\ell} + e^{-|n+m|a/\ell} \right]$$

Those in windows of width $\sim \ell$ centered at $n = \pm m$



⑥ $$\dot{u}(n, ma/c) = \frac{a}{10\ell} c \left[\frac{n-m}{|n-m|} e^{-|n-m|a/\ell} - \frac{(n+m)}{|n+m|} e^{-|n+m|a/\ell} \right]$$



Exercise 2

① $$g_\sigma(E) = \frac{1}{A}\sum_{\vec{k}}{}' \delta\left(E - \frac{\hbar^2 k^2}{2m} - \sigma\mu_B H\right)$$

$$\boxed{g_\sigma(E) = \frac{1}{2} g_0(E - \sigma\mu_B H)}$$

② $$n_\sigma = \int_{-\infty}^{+\infty} dE \; \frac{n}{2\epsilon_F} \frac{\theta(E-\sigma\mu_B H)}{e^{\beta(E-\mu)}+1}$$

$$= \int_{\sigma\mu_B H}^{\infty} dE \; \frac{n}{2\epsilon_F} \frac{1}{e^{\beta(E-\mu)}+1}$$

$$n_\sigma = \frac{n}{2\beta\epsilon_F}\left[-\ln\left(1+e^{-\beta(E-\mu)}\right)\right]_{\sigma\mu_B H}^{\infty}$$

$$\boxed{n_\sigma = \frac{n}{2\beta\epsilon_F} \ln\left[1 + e^{\beta(\mu - \sigma\mu_B H)}\right]}$$

③ $$\boxed{M = -\mu_B \frac{n}{2\beta\epsilon_F} \ln\left[\frac{1+e^{\beta(\mu-\mu_B H)}}{1+e^{\beta(\mu+\mu_B H)}}\right]}$$

④ $$n = \frac{n}{2\beta\epsilon_F} \ln\left\{\left[1+e^{\beta(\mu-\mu_B H)}\right]\left[1+e^{\beta(\mu+\mu_B H)}\right]\right\}$$

It is evident by inspection that if μ is solution for n, β, H it is also solution for $n, \beta, -H$!

(5) To linear order in H $\mu \simeq \mu_0$ and

$$M \simeq -\frac{\mu_B n}{2\beta\epsilon_F} \ln\left[\frac{1+e^{\beta\mu_0}(1-\beta\mu_B H)}{1+e^{\beta\mu_0}(1+\beta\mu_B H)}\right]$$

$$= -\frac{\mu_B n}{2\beta\epsilon_F} \ln\left[\frac{1+e^{\beta\mu_0}(1-\beta\mu_B H)}{1+e^{\beta\mu_0}} \frac{1}{1+\frac{e^{\beta\mu_0}}{1+e^{\beta\mu_0}}\beta\mu_B H}\right]$$

$$= -\frac{\mu_B n}{2\beta\epsilon_F} \ln\left[\left(1-\frac{e^{\beta\mu_0}}{1+e^{\beta\mu_0}}\beta\mu_B H\right)\left(1-\frac{e^{\beta\mu_0}}{1+e^{\beta\mu_0}}\beta\mu_B H\right)\right]$$

$$\simeq -\frac{\mu_B n}{2\beta\epsilon_F} \ln\left[1-\frac{2e^{\beta\mu_0}}{1+e^{\beta\mu_0}}\beta\mu_B H\right]$$

$$M \simeq \frac{\mu_B n}{\cancel{2}\cancel{\beta}\epsilon_F} \frac{\cancel{2}e^{\beta\mu_0}}{1+e^{\beta\mu_0}}\cancel{\beta}\mu_B H$$

$$= \frac{\mu_B^2 n}{\epsilon_F} \frac{e^{\beta\mu_0}}{1+e^{\beta\mu_0}} \cdot H$$

$$\boxed{\chi(T,\mu_0) = \frac{\mu_B^2 n}{\epsilon_F} \frac{e^{\beta\mu_0}}{1+e^{\beta\mu_0}}}$$

(6) $$e^{\beta\mu_0} = e^{\beta\epsilon_F}(1-e^{-\beta\epsilon_F}) = e^{\beta\epsilon_F}-1$$

$$\chi(T,\mu_0) = \chi(n,\epsilon_F,\beta) = \frac{\mu_B^2 n}{\epsilon_F} \frac{e^{\beta\epsilon_F}-1}{e^{\beta\epsilon_F}}$$

$$\boxed{\chi(n\,\epsilon_F\,\beta) = \frac{\mu_B^2 n}{\epsilon_F}\left[1-e^{-\beta\epsilon_F}\right]}$$