Bose in 2D $\qquad \epsilon_p = \frac{p^2}{2m}$

① $g(\epsilon) = \frac{1}{A} \sum_{\vec{p}} \delta\left(\epsilon - \frac{p^2}{2m}\right)$

$$= \frac{1}{A} \frac{A}{h^2} \int_0^\infty dp \, p \, 2\pi \, \delta\left(\epsilon - \frac{p^2}{2m}\right)$$

$$= \frac{2m\pi}{h^2} \int_0^\infty d\left(\frac{p^2}{2m}\right) \delta\left(\epsilon - \frac{p^2}{2m}\right)$$

$$\boxed{g(\epsilon) = \frac{2\pi m}{h^2} \theta(\epsilon)} \equiv c\,\theta(\epsilon)$$

② $\rho_n = \int_0^\infty d\epsilon \frac{g(\epsilon)}{e^{\beta(\epsilon-\mu)} - 1} = \frac{2\pi m}{h^2} \int_0^\infty d\epsilon \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$

$$= \frac{2\pi m}{h^2} k_B T \int_0^\infty dy \frac{1}{\frac{e^y}{z} - 1} = \frac{1}{\lambda^2} \int_0^\infty dy \frac{z e^{-y}}{1 - z e^{-y}}$$

$$= \frac{1}{\lambda^2} \int_0^z d(z e^{-y}) \frac{1}{1 - z e^{-y}} \qquad t \equiv z e^{-y}$$

$$\rho_n = \frac{1}{\lambda^2} \int_0^z dt \frac{1}{1-t} = \frac{1}{\lambda^2} \left[-\ln(1-t)\right]_0^z$$

$$\boxed{\rho_n = -\frac{1}{\lambda^2} \ln(1-z) \quad 0 \leq z \leq 1}$$

③ $$Z = \prod_{p} \frac{1}{1 - z e^{-\beta \epsilon_p}}$$

$$\beta P A = \ln Z = -\sum_{p} \ln\left(1 - z e^{-\beta \epsilon_p}\right)$$

$$= -A \int_0^\infty dE\, g(E) \ln\left(1 - z e^{-\beta E}\right)$$

$$= -\frac{2\pi m k_B T}{h^2} A \int_0^\infty dy \ln\left(1 - z e^{-y}\right)$$

$$= \frac{A}{\lambda^2} \sum_{n=1}^{\infty} \frac{z^n}{n} \int_0^\infty dy\, e^{-ny} = \frac{A}{\lambda^2} \sum_{n=1}^{\infty} \frac{z^n}{n^2}$$

$$\beta P \cancel{A} = \frac{\cancel{A}}{\lambda^2} g_2(z)$$

④ $$e^{-\lambda^2 \rho} = 1 - z \Rightarrow z = 1 - e^{-\lambda^2 \rho}$$

$$\lambda^2 \rho = \alpha$$

$$\alpha \ll 1 \qquad z = 1 - \left(1 - \alpha + \frac{\alpha^2}{2} + \cdots\right)$$

$$= \alpha - \frac{\alpha^2}{2} + \cdots = \lambda^2 \rho - \frac{(\lambda^2 \rho)^2}{2} + \cdots$$

$$\beta P = \frac{1}{\lambda^2}\left[z + \frac{z^2}{4} + \cdots\right]$$

$$= \frac{1}{\lambda^2}\left[\lambda^2 \rho - \frac{(\lambda^2 \rho)^2}{2} + \frac{1}{4}\left(\lambda^2 \rho - \frac{\lambda^2 \rho}{2} + \cdots\right)^2 + \cdots\right]$$

$$= \frac{1}{\lambda^2}\left[\lambda^2 \rho - \frac{(\lambda^2 \rho)^2}{4} + \cdots\right] = \rho\left[1 - \frac{\lambda^2 \rho}{4}\right]$$

$$\beta P = \rho\left[1 - \frac{\lambda^2 \rho}{4} + \cdots\right]$$

ELETTRONI NEL GRAFENE

① $$g(\epsilon) = \frac{1}{A} \sum_{\vec{p}} \delta(\epsilon - \epsilon_{\vec{p}}) \qquad \epsilon_{\vec{p}} = \alpha p$$

$$= \frac{1}{A} \frac{A}{h^2} \int_0^\infty dp\, p\, 2\pi\, \delta(\epsilon - \alpha p) \qquad \begin{array}{l} \epsilon = \alpha p \\ p = \frac{\epsilon}{\alpha} \end{array}$$

$$= \frac{1}{h^2} 2\pi \frac{1}{\alpha^2} \int_0^\infty d\epsilon\, \epsilon\, \delta(E - \epsilon)$$

$$\boxed{g(\epsilon) = \frac{2\pi}{\alpha^2 h^2} E\,\theta(\epsilon)} \equiv C E\,\theta(\epsilon)$$

② $$\rho = \int_0^\infty dE \frac{g(\epsilon)}{e^{\beta(\epsilon-\mu)}+1} = C \int_0^\infty dE \frac{E}{\frac{e^{\beta\epsilon}}{z}+1}$$

$$= C(k_B T)^2 \int_0^\infty dy\, y \frac{e^{-y} z}{1+z e^{-y}}$$

$$= C(k_B T)^2 \int_0^\infty dy\, y\, z e^{-y} \sum_{n=0}^\infty (-z e^{-y})^n$$

$$= C(k_B T)^2 \int_0^\infty dy\, y \sum_{n=1}^\infty (+z e^{-y})^n (-)^{n-1}$$

$$= C(k_B T)^2 \sum_{n=1}^\infty (-1)^{n-1} z^n \int_0^\infty dy\, y\, e^{-ny}$$

$$= C(k_B T)^2 \sum_{n=1}^\infty (-1)^{n-1} z^n \left(-\frac{\partial}{\partial n}\right) \int_0^\infty dy\, e^{-ny}$$

$$\rho = \frac{2\pi}{\alpha^2 h^2} (k_B T)^2 \sum_{n=1}^\infty (-1)^{n-1} \frac{z^n}{n^2} \equiv \frac{2\pi (k_B T)^2}{\alpha^2 h^2} f_2(z)$$

③ $$Z = \prod_\alpha \left(1 + z e^{-\beta \epsilon_\alpha}\right)$$

$$\beta P = \frac{1}{A} \ln Z = \frac{1}{A} \sum_{\vec{p}} \ln\left(1 + z e^{-\beta \epsilon_p}\right)$$

$$= \int_0^\infty dE \, g(E) \ln\left(1 + z e^{-\beta E}\right)$$

$$= C' (k_B T)^2 \int_0^\infty dy \, y \ln\left(1 + z e^{-y}\right)$$

$$= C' (k_B T)^2 \sum_{n=1}^\infty \frac{z^n}{n} \int_0^\infty dy \, y \, e^{-ny} (-1)^{n-1}$$

$$= C' (k_B T)^2 \sum_{n=1}^\infty (-1)^n \frac{z^n}{n} \left(-\frac{\partial}{\partial n}\right) \int_0^\infty dy \, e^{-ny}$$

$$\beta P = \frac{2\pi}{\alpha^2 h^2} (k_B T)^2 f_3(z)$$

$$f_3(z) = \sum_{n=1}^\infty (-1)^{n-1} \frac{z^n}{n^3}$$

④ $$l^2 = \frac{\alpha^2 h^2}{2\pi (k_B T)^2}$$

$$\rho l^2 \ll 1$$

$$\rho l^2 = z - \frac{z^2}{2^2} + \frac{z^3}{3^2} + \cdots \qquad z = a \rho l^2 + b (\rho l^2)^2 + \cdots$$

$$\rho l^2 = a (\rho l^2) + b (\rho l^2)^2 - \frac{a^2}{4} (\rho l^2)^2 + \cdots$$

$$\Rightarrow \quad a = 1 \qquad b = \frac{1}{4} \qquad z \simeq \rho l^2 + \frac{1}{4} (\rho l^2)^2 + \cdots$$

$$\beta P = \frac{1}{\ell^2}\left(z - \frac{z^2}{8} + \dots\right)$$

$$= \frac{1}{\ell^2}\left(\rho\ell^2 + \frac{1}{4}(\rho\ell^2)^2 - \frac{1}{8}(\rho\ell^2)^2 + \dots\right)$$

$$= \frac{1}{\ell^2}\left(\rho\ell^2 + \frac{1}{8}(\rho\ell^2)^2\right)$$

$$\beta P = \rho\left[1 + \frac{\rho\ell^2}{8} + \dots\right]$$