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$$\mathcal{H} = \sum_{i=1}^{N} h_i^{(1)} \qquad h^{(1)} = cp$$

$$Q_N = \frac{1}{N!} q^N \qquad q = \frac{1}{h^3} \int d\bar{p} \int d\bar{r}\, e^{-\beta h^{(1)}}$$

$$q = \frac{1}{h^3} \int d\bar{p}\, e^{-\beta c p} \int_V d\bar{r} = V \frac{4\pi}{h^3} \int_0^\infty dp\, p^2 e^{-\beta c p}$$

$$q = \frac{4\pi V}{(h\beta c)^3} \int_0^\infty dy\, y^2 e^{-y}$$

$$\int_0^\infty dy\, y^2 e^{-y} = \left[\frac{\partial^2}{\partial \alpha^2} \int_0^\infty dy\, e^{-\alpha y}\right]_{\alpha=1} = \frac{\partial^2}{\partial \alpha^2} \frac{1}{\alpha}\Big|_{\alpha=1} = 2$$

$$q = \frac{8\pi V}{(h\beta c)^3} = \frac{\pi V}{\ell^3} \qquad \ell^3 = \left(\frac{h\beta c}{2}\right)^3 \frac{1}{\pi} = \left(\frac{hc}{2k_B T}\right)^3 \frac{1}{\pi}$$

① $$\boxed{Q_N = \frac{1}{N!}\left(V\pi \left(\frac{2k_B T}{hc}\right)^3\right)^N}$$

② $$A = -N k_B T \ln\left[\frac{e\pi V}{N}\left(\frac{2k_B T}{hc}\right)^3\right]$$

③ $$E = \frac{\partial \beta A}{\partial \beta} = -\frac{\partial}{\partial \beta}\left[N \ln \frac{1}{\beta^3}\right] = 3 N k_B T$$

$$\boxed{E = 3 N k_B T} \qquad \boxed{S = -\frac{\partial A}{\partial T} = N k_B \ln\left[\frac{e\pi V}{N}\left(\frac{2k_B T}{hc}\right)^3\right] + 3 N k_B}$$

④ $$\boxed{C_V = 3 N k_B} \qquad C_V = T \frac{\partial S}{\partial T} = T N k_B \frac{3}{T} = 3 N k_B$$

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$$\mathcal{H} = \sum_{i=1}^{N} h_i^{(1)} \qquad h^{(1)} = \frac{p^2}{2m} + \frac{K}{2}\left[x^2 + \epsilon(\alpha x^3 + \gamma x^4)\right]$$

$$\boxed{Q_N = \frac{1}{N!} q^N}, \qquad q = \frac{1}{h}\int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dx \, e^{-h^{(1)}}$$

$$q = \frac{1}{h}\int_{-\infty}^{\infty} dp \, e^{-\frac{\beta p^2}{2m}} \cdot \int_{-\infty}^{\infty} dx \, e^{-\frac{\beta K}{2}[x^2 + \epsilon(\alpha x^3 + \gamma x^4)]}$$

$$\equiv q_1 \cdot q_2$$

$$\boxed{q_1 = \frac{1}{h}\int_{-\infty}^{\infty} dp \, e^{-\frac{\beta p^2}{2m}} = \sqrt{\frac{2\pi k_B T m}{h^2}} \equiv \frac{1}{\lambda}}$$

$$q_2 = \int_{-\infty}^{\infty} dx \, e^{-\frac{\beta K}{2} x^2} e^{-\frac{\beta K}{2}\epsilon(\alpha x^3 + \gamma x^4)}$$

$$\simeq \int_{-\infty}^{\infty} dx \, e^{-\beta \frac{K}{2} x^2}\left[1 - \frac{\beta K}{2}\epsilon\alpha x^3 - \frac{\beta K}{2}\epsilon\gamma x^4\right]$$

$$= \int_{-\infty}^{\infty} dx \, e^{-\beta \frac{K}{2} x^2}\left[1 - \frac{\beta K}{2}\epsilon\gamma x^4\right] \qquad \boxed{y = \sqrt{\frac{K}{2 k_B T}}}$$

$$q_2 = \sqrt{\frac{2 k_B T}{K}} \int_{-\infty}^{\infty} dy \left[1 - \frac{2}{\beta K}\epsilon\gamma y^4\right] e^{-y^2} \qquad \boxed{\ell = \sqrt{\frac{2\pi k_B T}{K}}}$$

$$\boxed{q_2 = \sqrt{\frac{2 k_B T}{K}}\left[\sqrt{\pi} - \frac{2}{\beta K}\epsilon\gamma\frac{3}{4}\sqrt{\pi}\right] = \ell\left[1 - \frac{3}{4\pi}\gamma \ell^2 \epsilon\right]}$$

① $Q_N = \frac{1}{N!}(q_1 \cdot q_2)^N = \frac{1}{N!}\left(\frac{l}{\lambda}\right)^N \left(1 - \frac{3}{4\pi}\gamma l^2 \epsilon\right)^N$

$$\boxed{Q_N \simeq \left(\frac{e l}{N\lambda}\right)^N \left[1 - N\frac{3}{4\pi}\gamma l^2 \epsilon\right]}$$

② $A = -k_B T \ln Q_N \simeq -N k_B T \ln\left[\frac{e l}{\lambda}\left(1 - \frac{3}{4\pi}\gamma l^2 \epsilon\right)\right]$

$$= -N k_B T \left\{\ln\frac{e l}{N\lambda} + \ln\left(1 - \frac{3}{4\pi}\gamma l^2 \epsilon\right)\right\}$$

$$\boxed{A \simeq -N k_B T \ln\left(\frac{e l}{N\lambda}\right) + \left[N k_B T \frac{3}{4\pi}\gamma l^2\right]\epsilon}$$

③ $E = \frac{\partial}{\partial\beta}(\beta A) = \frac{\partial}{\partial\beta}\left[-N \ln\left(\frac{e}{N}\sqrt{\frac{m}{h^2 K} 2\pi k_B T}\right) + N\frac{3}{4\pi}\gamma \frac{2\pi k_B T}{K}\epsilon\right]$

$$E = \frac{\partial}{\partial\beta}\left[-N\left(\ln\frac{1}{\beta} - \frac{3}{4\pi}\frac{2\pi}{K}\gamma\epsilon\frac{1}{\beta}\right)\right]$$

$$\boxed{E = N\left(k_B T - \frac{3}{2}\frac{\gamma}{K}\epsilon (k_B T)^2\right)}$$

④

$$\boxed{C_V = \frac{\partial E}{\partial T} = N k_B\left[1 - \epsilon 3\frac{\gamma}{K}k_B T\right]}$$

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$$h^{(1)} = \frac{p^2}{2m} + \frac{K}{2}\left[x^2 + \epsilon(\alpha x^2 + \gamma x^4)\right]$$

$$q_2 \simeq \int_{-\infty}^{+\infty} dx\, e^{-\frac{\beta K}{2}x^2}\left[1 - \frac{\beta K}{2}\epsilon\alpha x^2 - \frac{\beta K}{2}\epsilon\gamma x^4\right]$$

Quindi q_2 ha un termine aggiuntivo

$$\Delta q_2 = -\int_{-\infty}^{+\infty} dx\, e^{-\frac{\beta K}{2}x^2}\frac{\beta K}{2}\epsilon\alpha x^2$$

$$= -\sqrt{\frac{2k_B T}{K}}\int_{-\infty}^{+\infty} dy\, e^{-y^2}\epsilon\alpha y^2 = -\sqrt{\frac{2k_B T}{K}}\frac{\sqrt{\pi}}{2}\epsilon\alpha$$

$$\Delta q_2 = -\frac{1}{2} l \epsilon\alpha, \quad q_2 = l\left[1 - \frac{\epsilon\alpha}{2} - \frac{3}{4\pi}\gamma l^2\epsilon\right]$$

$$\Rightarrow Q_N = \frac{1}{N!}\left(\frac{l}{\lambda}\right)^N\left(1 - \frac{\epsilon\alpha}{2} - \frac{3}{4\pi}\gamma l^2\epsilon\right)^{N}$$

$$A = -Nk_B T \ln\left(\frac{e\,l}{N\lambda}\right) + Nk_B T\left(\frac{\epsilon\alpha}{2} + \frac{3}{4\pi}\gamma l^2\epsilon\right)$$

$$E = \frac{\partial}{\partial\beta}\beta A = N\left(k_B T - \frac{3}{2}\frac{\gamma}{K}\epsilon(k_B T)^2\right)$$

$$C_V = \frac{\partial E}{\partial T} = Nk_B\left[1 - \epsilon\, 3\frac{\gamma}{K}k_B T\right]$$