Esercizio 1

① $2 \cdot \frac{2k_F}{(2\pi/L)} = \frac{2k_F}{\pi} L = N \Rightarrow \boxed{k_F = \frac{\pi}{2} n}$

② $T[n] = \int dx\, n(x)\, \frac{1}{3}\, \epsilon_F(n(x))$

$= \int dx\, n(x)\, \frac{1}{3}\, \frac{1}{2} \left[\frac{\pi}{2} n(x)\right]^2$

$\boxed{T[n] = \int dx\, \frac{\pi^2}{24} n^3(x)}$ $\quad \hbar = m = e = 1$!

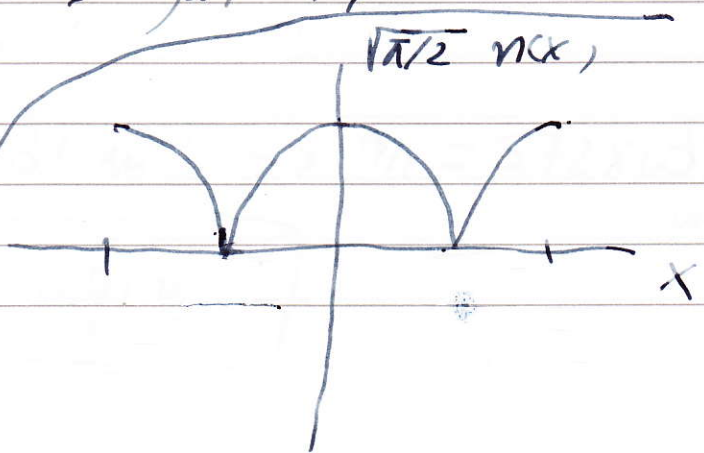
③ $E[n] = T[n] + \int dx\, n(x) v(x)$

$\mu = \frac{\delta E}{\delta n(x)} = \frac{\pi^2}{8} n^2(x) + v(x)$

④ $n(x) = \frac{2\sqrt{2}}{\pi} \sqrt{\mu - v(x)}$

⑤ $\mu - \frac{\pi}{4} \sin^2(x) \geq 0 \Rightarrow \mu \geq \pi/4$

$n(x) = \frac{\sqrt{2}}{\sqrt{\pi}} |\cos(x)|$



⑥

Esercizio 2

① $E_d = \frac{m_c}{m_e}\frac{1}{\epsilon^2} \cdot R_y = 0.015 \frac{1}{(17.88)^2} 13.6\ (eV)$

$= 6.38 \times 10^{-4}\ eV$

② $a_B^* = \frac{\epsilon}{m_c/m_e} \cdot a_B = \frac{17.88}{0.015} \times 0.529\ \text{Å} = 631\ \text{Å}$

③ $n_{cr} = \frac{1}{\frac{4\pi}{3} a_B^{*3}} = \frac{3}{4\pi (6{,}31 \times 10^{-6})^3}\ cm^{-3}$

$= 9.50 \times 10^{+14}\ cm^{-3}$

④ $n_i = 2.5 \left(\frac{m_c}{m_e}\frac{m_v}{m_e}\right)^{3/4} \left(\frac{T}{300K}\right)^{3/2} e^{-E_g/2k_BT} \times 10^{19}\ cm^{-3}$

$= 2.5 \left(0.015 \times 0.2\right)^{3/4} \left(\frac{1}{3}\right)^{3/2} e^{-1/(10 \times 8.62 \times 10^{-5} \times 100)} \times 10^{19}\ cm^{-3}$

$= 2.5 \times 1.28 \times 10^{-2} \times 0.192 \times 9.16 \times 10^{-6} \times 10^{19}\ cm^{-3}$

$n_i = 5.63 \times 10^{11}\ cm^{-3}$

⑤ $n_d >> n_{cr} = 3.5 \times 10^{14}\ cm^{-3} >> n_i = 5.72 \times 10^{11}\ cm^{-3}$

extrinsic regime

⑥ A metal, since the electrons on the donors unbind for $n_{D1} > n_{D2}$ and go into the conduction band, as stated at 3.

Note:

④ Eq 28.20 A.17.