SCRITTO DEL 18-01-10

Esercizio 1

①
$$Q_N = \frac{1}{h^{3N} N!}\left[\int d\bar{p}\, e^{-\frac{\beta \bar{p}^2}{2m}} \int d\bar{r}\, e^{-\beta\frac{K\bar{r}^2}{2}}\right]^N$$

$$= \frac{1}{h^{3N} N!}\left[\left(\int_{-\infty}^{+\infty} dt\, e^{-\frac{\beta t^2}{2m}}\right)^3 \left(\int_{-\infty}^{+\infty} dt\, e^{-\frac{\beta K}{2}t^2}\right)^3\right]^N$$

$$= \frac{1}{h^{3N} N!}\left[(2\pi m k_B T)^{3/2}\left(\frac{2k_B T\pi}{K}\right)^{3/2}\right]^N$$

$$= \frac{1}{N!}\left[\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2}\left(\frac{2\pi k_B T}{K}\right)^{3/2}\right]^N$$

$$Q_N = \frac{1}{N!}\left[\frac{2\pi k_B T}{h}\sqrt{\frac{m}{K}}\right]^{3N}$$

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②
$$-\frac{K}{\beta}\frac{\partial \ln Q_N}{\partial K} = -\frac{K}{\beta}\frac{\partial}{\partial K}\left[N \ln\left\{\int d\bar{p}\, e^{-\frac{\beta p^2}{2m}}\int d\bar{r}\, e^{-\frac{\beta K\bar{r}^2}{2}}\right\}\right.$$

$$\left. - \ln\left[h^{3N} N!\right]\right] = -\frac{NK}{\beta}\frac{\int d\bar{r}\left[-\frac{\beta\bar{r}^2}{2}\right]e^{-\beta\frac{K\bar{r}^2}{2}}}{\int d\bar{r}\, e^{-\frac{\beta K\bar{r}^2}{2}}}$$

$$= N\left\langle \frac{K\bar{r}^2}{2}\right\rangle = \left\langle \sum_{i=1}^{N} \frac{K\bar{r}_i^2}{2}\right\rangle = \langle U_{pot}\rangle$$

$$\langle U_{pot}\rangle = -\frac{K}{\beta}\frac{\partial}{\partial K}\left\{-\frac{3}{2}N\ln K + \text{cost}\right\}$$

$$= \frac{3}{2}N k_B T$$

③ In analogia con il punto precedente

$$m k_B T \frac{\partial}{\partial m} \ln Q_N =$$

$$= m k_B T \frac{\partial}{\partial m} \ln \left(\int d\bar{p}\, e^{-\frac{\beta \bar{p}^2}{2m}} \right)^N$$

$$= m k_B T N \int d\bar{p}\, \frac{\beta \bar{p}^2}{2m^2} e^{-\frac{\beta \bar{p}^2}{2m}} \Big/ \int d\bar{p}\, e^{-\frac{\beta \bar{p}^2}{2m}}$$

$$= N \left\langle \frac{\bar{p}^2}{2m} \right\rangle = \left\langle \sum_i \frac{p_i^2}{2m} \right\rangle = \langle T \rangle$$

$$\langle T \rangle = m k_B T \frac{\partial}{\partial m} \left\{ \frac{3}{2} N \ln m + \text{cost} \right\}$$

$$= \frac{3}{2} N k_B T$$

④

$$\mu = + \left.\frac{\partial A}{\partial N}\right|_{V,T} \qquad A = -k_B T \ln \left\{ \frac{1}{N!} \left(\frac{k_B T}{\hbar \omega} \right)^{3N} \right\}$$

$$\omega = \sqrt{K/m} \qquad A \simeq -k_B T N \ln \left(\frac{e}{N} \left[\frac{k_B T}{\hbar \omega} \right]^3 \right)$$

$$\mu = k_B T \left[1 + \ln \left\{ \frac{N}{e} \left(\frac{\hbar \omega}{k_B T} \right)^3 \right\} \right]$$

$$\mu = k_B T \ln \left[N \left(\frac{\hbar \omega}{k_B T} \right)^3 \right]$$

Esercizio 2

① $$Z_1 = \sum_{\sigma=\pm1} e^{-\beta E_\sigma} = 2\cosh(\beta\mu_B H)$$

$$Q_N = \left[2\cosh(\beta\mu_B H)\right]^N$$

$$A = -N k_B T \ln\left[2\cosh(\beta\mu_B H)\right]$$

② $$S = -\frac{\partial A}{\partial T} = N k_B \Big\{ \ln\left[2\cosh(\beta\mu_B H)\right]$$

$$+ T \,\mathrm{tgh}(\beta\mu_B H)\left[-\frac{\beta\mu_B H}{T}\right]\Big\}$$

$$S = N k_B \left\{ \ln\left[2\cosh(\beta\mu_B H)\right] - \beta\mu_B H \,\mathrm{tgh}(\beta\mu_B H)\right\}$$

$$k_B T \ll \mu_B H \Rightarrow \beta\mu_B H \gg 1$$

$$S \simeq N k_B \left\{ \ln e^{\beta\mu_B H} - \beta\mu_B H \right\} \simeq 0$$

③ $$M = -\frac{1}{V}\frac{\partial A}{\partial H} = \frac{N}{V} \cancel{k_B T}\left[\mathrm{tgh}(\beta\mu_B H)\right]\cancel{\beta}\mu_B$$

$$= \frac{N\mu_B}{V}\,\mathrm{tgh}(\beta\mu_B H)$$

④ $$\chi = \frac{\partial M}{\partial H}\Big|_{H=0} = \frac{N\mu_B}{V}\frac{1}{\cosh^2(\beta\mu_B H)}\beta\mu_B\Big|_{H=0}$$

$$\chi = \frac{N\mu_B^2}{V k_B T}$$