Exercice 2 21/06/13

1 From Chapt. 2 A&M

$$\gamma = \frac{\pi^2}{3} k_B^2 g(\epsilon_F) \;,\; g(\epsilon_F) = \frac{3\gamma}{\pi^2 k_B^2}$$

In order to have γ^* per mole of atoms of the metal, if Z is the valence of the atoms and n_a the density of atoms

$$\gamma^* = \gamma \frac{N_A Z}{n_a Z} = \gamma \frac{N_A}{n} Z = \gamma \frac{N}{n} = \gamma V_{mole}$$

since the electronic density is $n = Z n_a$.

Therefore $\gamma = \gamma^* \frac{n}{N_A Z} = \frac{\gamma^*}{V_{mole}}$

$$\gamma(cm^{-3} eV K^{-2}) = \gamma^*(cal\ mole^{-1} K^{-2}) \times \frac{n}{N_A Z} \times 4.13 \times 10^7 \times \frac{1}{1.6 \times 10^{-12}}$$

$T_c = 3.72\,°K$ holds for Sn, with $n = 14.8 \times 10^{22}\ cm^{-3}$ and $Z = 4$

$$\gamma = 7.08 \times 10^{14}\ cm^{-3}\ eV\ K^{-2}$$

$$g(\epsilon_F) = \frac{3 \times 7.08 \times 10^{+14}}{\pi^2 (8.62 \times 10^{-5})^2} = 2.90 \times 10^{22}\ cm^{-3}\ eV^{-1}$$

② $\omega_D = \frac{k_B \Theta}{\hbar} = \frac{1.38\times10^{-16}\times170}{1.05\times10^{-27}} = 2.23\times10^{13}\,s^{-1}$

③ $\exp\left[-\frac{1}{N_0V_0}\right] = \frac{k_B T_c}{1.13\hbar\omega_D} = \frac{T_c}{1.13\,\Theta} = \frac{3.72}{170\times1.13} = 1.94\times10^{-2}$

$(N_0V_0)^{-1} = -\ln(1.94\times10^{-2}) = 3.94$

$N_0 = \frac{g(\epsilon_F)}{2} = 1.43\times10^{22}\ cm^{-3}\,eV^{-1}$

$N_0V_0 = 0.254$

$V_0 = 0.254/(1.43\times10^{22})\ cm^3\,eV = 1.78\times10^{-23}\ cm^3\,eV$

④ $\Delta(0) = 1.76\,k_BT_c = 3.72\times1.76\times8.62\times10^{-5}\ eV$

$= 5.64\times10^{-4}\ eV$

⑤ $g(\epsilon_F) = \frac{3}{2}\frac{n}{\epsilon_F} = \frac{3}{\cancel{2}}\frac{n}{\frac{\hbar^2}{\cancel{2}m^*}(3\pi^2n)^{2/3}} = \frac{3n^{\frac{1}{3}}m^*}{\hbar^2(3\pi^2)^{2/3}}$

$n = \left(\frac{\hbar^2(3\pi^2)^{2/3}g(\epsilon_F)}{3m^*}\right)^3$

$= \left(\frac{(1.05\times10^{-27})^2(3\pi^2)^{2/3}\,2.90\times10^{22}}{1.32\times 3\times9.11\times10^{-28}\times1.6\times10^{-12}}\right)^3$

$= \frac{3.43}{(1.32)^3}\times10^{23}\ cm^{-3}$ $=1.49\times(10)^{23}$

⑥ $\xi \sim \frac{\epsilon_F}{k_F\Delta} = \frac{3}{2}\frac{n}{g(\epsilon_F)}\frac{1}{(3\pi^2n)^{\frac{1}{3}}\Delta} = \frac{3}{2}\frac{n^{2/3}}{g(\epsilon_F)\Delta(3\pi^2)^{\frac{1}{3}}}$

$= \cancel{1.45}\ 0.269\times10^{-4}\ cm\ .\ = \cancel{1450}\ 2690\ \text{Å}$

Exercise 1 21/06/13 ①

① $U = \frac{1}{2} \sum_n \left\{ K_1 [u(n) - u(n+1)]^2 + K_2 [u(n) - u(n+2)]^2 \right\}$

② $M \ddot{u}(m) = - \frac{\partial U}{\partial u(m)} = - K_1 [2u(m) - u(m+1) - u(m-1)]$

$- K_2 [2u(m) - u(m+2) - u(m-2)]$

③

$-\omega^2 M \epsilon e^{i(qna - \omega t)} = -K_1 \epsilon e^{i(qna - \omega t)} \times$

$\times [2 - e^{iqa} - e^{-iqa}] - K_2 \epsilon e^{i(qna - \omega t)} \times$

$\times [2 - e^{2iqa} - e^{-2iqa}]$

$\Rightarrow M\omega^2 = 4K_1 \sin^2(\frac{qa}{2}) + 4K_2 \sin^2(qa)$

$$\boxed{\omega^2 = \frac{4K_1}{M} \left[\sin^2(\frac{qa}{2}) + \frac{K_2}{K_1} \sin^2(qa) \right]}$$

④ $q \to 0$

$\omega^2(q) \simeq \frac{4K_1}{M} \left[\frac{q^2 a^2}{4} + \frac{K_2}{K_1} q^2 a^2 \right]$

$\omega(q) = \sqrt{\frac{4K_1}{M} \frac{a^2}{4}} \sqrt{1 + 4\frac{K_2}{K_1}} \cdot q$

$$\boxed{c = a \sqrt{\frac{K_1}{M}} \sqrt{1 + 4\frac{K_2}{K_1}}}$$

(5) $$\omega^2(q) = \frac{4K_1}{M}\left[\sin^2\left(\frac{qa}{2}\right) + \frac{1}{2}\sin^2(qa)\right]$$

For $q > 0$, $q \leq \frac{\pi}{a}$ $0 < \frac{qa}{2} \leq \frac{\pi}{2}$,

hence $\sin^2\left(\frac{qa}{2}\right) > 0 \Rightarrow \omega^2(q) > 0$, $\omega(q) > 0$!

(6) Since $\frac{d\omega^2(q)}{dq} = \left(\frac{d\omega}{dq}\right)\omega(q)$ and

$\omega(q) > 0$ for $0 < q \leq \frac{\pi}{a}$, in the

same range a zero of $\frac{d\omega}{dq}$ implies

a zero in $\frac{d\omega^2}{dq}$.

$$\frac{d\omega^2(q)}{dq} = \frac{4K_1}{M}\left[\frac{a}{2}\cdot 2\sin\frac{qa}{2}\cos\frac{qa}{2} + \frac{1}{2}a\, 2\sin qa\cos qa\right]$$

$$= \frac{4K_1}{M}\frac{a}{2}\left[\sin qa + 2\sin qa\cos qa\right]$$

$$\frac{d\omega^2}{dq} = \frac{2K_1 a}{M}\sin qa\left[1 + 2\cos qa\right]$$

In $0 < q < \frac{\pi}{a}$ $\sin qa > 0$, therefore

$$\frac{d\omega^2}{dq} = 0 = \frac{d\omega}{dq} \Rightarrow \cos qa = -\frac{1}{2}$$

Therefore $q_1 = \frac{2\pi}{3a}$, $q_2 = \frac{4\pi}{3a}$; only q_1 is in the range $0 < q < \frac{\pi}{a}$

Also $\frac{d^2\omega(q)}{dq} < 0 \Rightarrow \frac{d\omega(q)}{dq} < 0$ in the range of interest.

Therefore