Esercizio 1. 17/11/11

1. $Q_N(V,T) = \frac{q^N}{N!}$, $q = \frac{1}{h^3}\int_V d\bar{r}\int d\bar{p}\, e^{-\beta h(p)}$

$$q = \frac{V}{h^3}\int_{-\infty}^{\infty} dp_x\, e^{-\frac{\beta p_x^2}{2m_x}} \int_{-\infty}^{\infty} dp_y\, e^{-\frac{\beta p_y^2}{2m_y}} \int_{-\infty}^{+\infty} dp_z\, e^{-\frac{\beta p_z^2}{2m_z}}$$

$$\frac{1}{h}\int_{-\infty}^{+\infty} dp_\alpha\, e^{-\frac{\beta p_\alpha^2}{2m_\alpha}} = \frac{1}{h}\sqrt{\frac{\pi}{\beta/2m_\alpha}} = \sqrt{\frac{2\pi m_\alpha k_B T}{h^2}}$$

$$q = V\sqrt{\frac{2\pi m_x k_B T}{h^2}}\sqrt{\frac{2\pi m_y k_B T}{h^2}}\sqrt{\frac{2\pi m_z k_B T}{h^2}} \equiv \frac{V}{\lambda^3}$$

- $Q_N(V,T) = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$

2.

$$Z(\mu,V,T) = \sum_{N=0}^{\infty} e^{\beta\mu N} Q_N = \sum_{N=0}^{\infty}\frac{1}{N!}\left[\frac{e^{\beta\mu}V}{\lambda^3}\right]^N$$

$$= \exp\left[\frac{e^{\beta\mu}V}{\lambda^3}\right]$$

3. $\ln Z = \beta P V = \frac{e^{\beta\mu}V}{\lambda^3}$

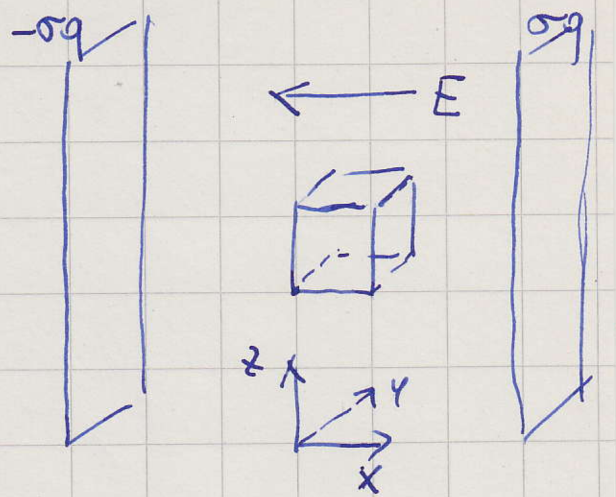
$$e^{\beta\mu} = \lambda^3\beta P \Rightarrow \mu = k_B T \ln\left[\frac{P\lambda^3}{k_B T}\right]$$

4. $A(N,V,T) = -k_B T \ln\left[\frac{eV}{N\lambda^3}\right]^N = -N k_B T \ln\left[\frac{eV}{N\lambda^3}\right]$

$$\mu = \frac{\partial A}{\partial N} = -k_B T \ln\frac{eV}{N\lambda^3} + k_B T = k_B T \ln\frac{N\lambda^3}{V}$$

$$\Rightarrow k_B T \ln\frac{P\lambda^3}{k_B T} = k_B T \ln\frac{N\lambda^3}{V} \Rightarrow P = \frac{N}{V}k_B T = \rho k_B T$$

Esercizio 2. 17/11/11



1. $E = -4\pi\sigma q$

$$V(x) = -\int_0^x dx\, E(x) = 4\pi\sigma q\, x$$

$$\boxed{h(\bar{p}) = \frac{p^2}{2m} + 4\pi\sigma q^2 x}$$

$$Q_N(V,T) = \frac{1}{N!}\, q^N \quad , \quad q = \frac{1}{h^3}\int_V d\bar{r}\int d\bar{p}\, e^{-h(p)\beta}$$

$$q = \frac{1}{h^3}\int d\bar{p}\, e^{-\beta\frac{p^2}{2m}} \int_0^{L_x} dx\, e^{-\beta 4\pi\sigma q^2 x}\int_0^{L_y} dy \int_0^{L_z} dz$$

$$= \frac{1}{\lambda^3} L_y L_z \int_0^{L_x} dx\, e^{-x/\ell} \qquad \ell = \frac{K_B T}{4\pi\sigma q^2}$$

$$q = \frac{L_y L_z}{\lambda^3}\left\{-\frac{1}{1/\ell}\, e^{-x/\ell}\right\}_0^{L_x} = \frac{L_y L_z \ell}{\lambda^3}\left[1 - e^{-L_x/\ell}\right]$$

- $$Q_N = \frac{1}{N!}\left[\frac{L_y L_z \ell}{\lambda^3}\left(1 - e^{-L_x/\ell}\right)\right]^N$$

- $$A(N,V,T) = -K_B T \ln\left\{\left[\frac{e}{N}\frac{L_y L_z \ell}{\lambda^3}\left(1 - e^{-L_x/\ell}\right)\right]^N\right\}$$

$$A(N,V,T) = -N K_B T \ln\left[\frac{e}{N}\frac{L_y L_z \ell}{\lambda^3}\left(1 - e^{-L_x/\ell}\right)\right]$$

2. $\delta V = L_y L_z\, \delta L_x$

$$P = -\frac{\partial A}{\partial V} = -\frac{1}{L_y L_z}\frac{\partial A}{\partial L_x}$$

$$P = -\frac{1}{L_y L_z} \frac{\partial}{\partial L_x}\left\{-N k_B T \ln\left(1 - e^{-L_x/\ell}\right)\right\}$$

$$= \frac{N k_B T}{L_y L_z} \frac{(1/\ell)\, e^{-L_x/\ell}}{1 - e^{-L_x/\ell}}$$

$$\bullet \quad P = \frac{N k_B T}{\ell L_y L_z} \frac{e^{-L_x/\ell}}{1 - e^{-L_x/\ell}}$$

2. $\delta V = L_x L_y \delta L_z$

$V = L_x L_y L_z$

$$P = -\frac{\partial A}{\partial V} = -\frac{1}{L_x L_y}\frac{\partial A}{\partial L_z}$$

$$= -\frac{1}{L_x L_y}\frac{\partial}{\partial L_z}\left\{-N k_B T \ln L_z\right\} = \frac{N k_B T}{L_x L_y L_z} = \frac{N k_B T}{V}$$

$$\bullet \quad P = \frac{N k_B T}{L_x L_y L_z} = \frac{N k_B T}{V}$$

4. $$\rho(x) = N\langle \delta(\bar{r} - \bar{r}_1)\rangle = N \frac{\int d\bar{r}_1\, e^{-\beta 4\pi\sigma q^2 x_1}\, \delta(\bar{r} - \bar{r}_1)}{\int d\bar{r}_1\, e^{-\beta 4\pi\sigma q^2 x_1}}$$

$$= \frac{e^{-x/\ell}}{\left(1 - e^{-L_x/\ell}\right)} \cdot \frac{N}{\ell L_y L_z} = \frac{N}{V}\frac{L_x}{\ell}\frac{e^{-x/\ell}}{1 - e^{-L_x/\ell}}$$

$$P = \frac{N k_B T}{L_y L_z}\frac{1}{\ell}\frac{e^{-L_x/\ell}}{1 - e^{-L_x/\ell}} = k_B T\, \rho(L_x)$$

$$P = \frac{N k_B T}{L_x L_y L_z} = k_B T\, \rho(0)\, \frac{\ell\left(1 - e^{-L_x/\ell}\right)}{L_x}$$