# COMPITO DEL 18-12-07

## Esercizio 1 : Fermioni ideali in 2D

(1)

$$g(E) = \frac{2}{A} \sum_{\vec{p}} \delta\left(E - \frac{p^2}{2m}\right)$$

$$= \frac{2}{A} \frac{A}{h^2} \int_0^\infty dp\, p \cdot 2\pi\, \delta\left(E - \frac{p^2}{2m}\right)$$

$$= \frac{4\pi}{h^2} m \int_0^\infty d\left(\frac{p^2}{2m}\right) \delta\left(E - \frac{p^2}{2m}\right)$$

$$\boxed{g(E) = \frac{4\pi m}{h^2} \theta(E)} = \frac{m}{\pi \hbar^2} \theta(E)$$

(2) $$\frac{\langle N \rangle}{A} \equiv \rho = \int_0^\infty dE \frac{g(E)}{e^{\beta(E-\mu)}+1} = \frac{m k_B T}{\pi \hbar^2} \int_0^\infty dt \frac{1}{\frac{e^t}{z}+1}$$

$$= 2 \cdot \frac{2\pi m k_B T}{h^2} \int_0^\infty dt\, z e^{-t} / (1 + z e^{-t})$$

$$= \frac{2}{\lambda^2} \int_0^z d(z e^{-t}) / (1 + z e^{-t}) \qquad y = z e^{-t}$$

$$\boxed{\rho = \frac{2}{\lambda^2} \ln(1+z)} \qquad \boxed{\beta \epsilon_F = \ln(1 + e^{\beta\mu})}$$

(3)

$$\beta P = \frac{\ln Z}{A} = \frac{2\cdot}{A}\sum_{p} \ln\left(1+e^{-\beta(\epsilon_p-\mu)}\right)$$

$$= \int_0^\infty dE\, g(E)\, \ln\left(1+e^{-\beta(E-\mu)}\right)$$

$$= \frac{4\pi m}{h^2}\int_0^\infty dE\, \ln\left(1+e^{-\beta(E-\mu)}\right)$$

$$= \frac{2}{\lambda^2}\int_0^\infty dt\, \ln\left(1+ze^{-t}\right) = \frac{2}{\lambda^2} f_2(z)$$

$$\beta P = \frac{2}{\lambda^2} f_2(z) \; ; \qquad f_2(z) = \sum_{n=1}^{\infty} (-1)^{n-1}\frac{z^n}{n^2}, \quad z<1.$$

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(4) $\rho\lambda^2 \to 0$ $\qquad y = \lambda^2\rho/2$

$$z = e^{\frac{\rho\lambda^2}{2}} - 1 = e^{y} - 1 = y + \frac{y^2}{2} + \frac{y^3}{6} + \cdots$$

$$\beta P = \frac{2}{\lambda^2}\left[z - \frac{z^2}{4} + \frac{z^3}{9} + \cdots\right]$$

$$= \frac{2}{\lambda^2} z\left[1 - \frac{z}{4} + \frac{z^2}{9} + \cdots\right]$$

$$= \frac{2}{\lambda^2} y\left[1 + \frac{y}{2} + \frac{y^2}{6} + \cdots\right]\left[1 - \frac{1}{4}\left(y + \frac{y^2}{2} + \cdots\right) + \frac{y^2}{9} + \cdots\right]$$

$$= \rho\left[1 + y\left(\frac{1}{2} - \frac{1}{4}\right) + y^2\left(-\frac{1}{8} + \frac{1}{6} - \frac{1}{8} + \frac{1}{9}\right)\right]$$

$$\frac{\beta P}{\rho} = 1 + \frac{\rho\lambda^2}{8} + \frac{1}{144}(\rho\lambda^2)^2 + \cdots$$

## Esercizio 2: Bosoni ideali in 4D

(1)

$$g(E) = \frac{1}{V} \sum_{\vec{p}} \delta\left(E - \frac{p^2}{2m}\right)$$

$$= \frac{1}{V} \frac{V}{h^4} 2\pi^2 \int_0^\infty dp\, p^3\, \delta\left(E - \frac{p^2}{2m}\right)$$

$$= \frac{2\pi^2}{h^4} \left[\frac{p^3}{p/m}\right]_{p=\sqrt{2mE}} = \frac{2\pi^2 m}{h^4} 2mE$$

$$= 4\frac{\pi^2 m^2}{h^4} E$$

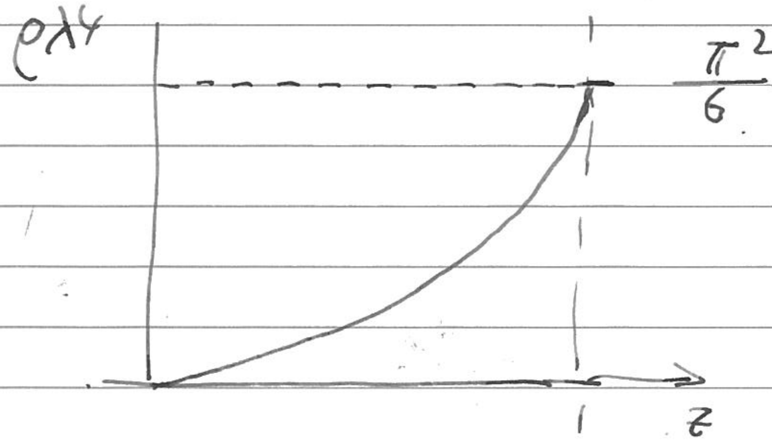
(2)

$$\frac{\langle N \rangle_n}{V} = \rho_n = \int_0^\infty dE \frac{g(E)}{e^{\beta(E-\mu)} - 1} = 4\frac{\pi^2 m^2}{h^4} \int_0^\infty dE \frac{E}{\frac{e^{\beta E}}{z} - 1}$$

$$= 4\frac{\pi^2 m^2 (k_B T)^2}{h^4} \int_0^\infty dt \frac{t z e^{-t}}{1 - z e^{-t}} \qquad \boxed{\rho = \rho_n + \rho_s}$$

$$\rho_n = \frac{1}{\lambda^4} g_2(z) = \sum_{n=1}^\infty \frac{z^n}{n^2} \; ; \; \rho_s = \frac{1}{V} \frac{z}{1-z}$$

(3)




$$y_c = (\lambda^4 \rho)_c = \frac{\pi^2}{6}$$

④ $(\lambda^4\rho)_c = g_2(1)$

$$\rho_s = \rho - \rho_n = \rho - \frac{g_2(2)}{\lambda^2}$$

$\pi < \pi_c$

$$\rho_s = \rho - \frac{g_2(1)}{\lambda^2} = \rho - \frac{g_2(1)}{\lambda^2\rho}\rho$$

$$= \rho\left[1 - \frac{g_2(1)}{\lambda^2\rho}\right] = \rho\left[1 - \frac{(\lambda^4\rho)_c}{\lambda^4\rho}\right]$$

$$= \rho\left[1 - \left(\frac{\pi}{\pi_c}\right)^2\right]$$

$$\boxed{\frac{\rho_s}{\rho} = 1 - \left(\frac{\pi}{\pi_c}\right)^2}$$