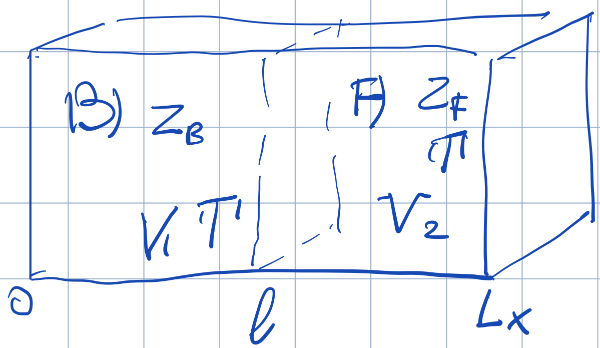
ESERCIZIO 1, 16.12.22



1)

Fermioni)

$$\beta P_F = \frac{1}{V_2} \ln\left[Z(V_2, T', z_F)\right] = \frac{g_s}{\lambda_{T'}^3} f_{5/2}(z_F)$$

$$\boxed{g_s = 2}$$

$$f_\alpha(z) = \sum_{n=1}^{\infty} (-1)^n \frac{z^n}{n^\alpha} \quad , \quad z<1$$

$$\boxed{\lambda_{T'}^3 = \frac{h^3}{(2\pi m k_B T')^{3/2}}}$$

Bosoni)

$$\beta P_B = \frac{1}{V_1} \ln\left[Z(V_1, T', z_B)\right] = \frac{1}{\lambda_{T'}^3} g_{5/2}(z_B)$$

$$g_\alpha(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^\alpha}$$

2)

$$\rho = \frac{\langle N \rangle}{V} = z \frac{\partial}{\partial z}\left[\frac{1}{V} \ln\left[Z(V, T', z)\right]\right]\Bigg|_{V, T'}$$

F) $$\rho_F = \frac{g_s}{\lambda_{T'}^3} f_{3/2}(z_F) \quad \left[z \frac{d\, f_\alpha(z)}{dz} = f_{\alpha-1}(z)\right]$$

B) $$\rho_B = \frac{1}{\lambda_{T'}^3} g_{3/2}(z_B) \quad \left[z \frac{d\, g_\alpha(z)}{dz} = g_{\alpha-1}(z)\right]$$

③ Essendo il setto di separazione mobile, l'equilibrio meccanico richiede $P_F = P_B$.

Dalla prima figura allegata si desume che per $z_B = 1$

$$\beta P_B \lambda_{T}^3 = g_{5/2}(1) = 1.34 \pm 0.01 = f_{5/2}(z_F) = 2 f_{5/2}(0.75 \pm 0.01)$$

$$\Rightarrow \boxed{z_F = 0.75 \pm 0.01}$$

④ Dalla seconda figura si desume

$$\lambda_{T}^3 \rho_B = g_{3/2}(1) = 2.60$$

$$\lambda_{T}^3 \rho_F = f_{3/2}(0.75 \pm 0.01) = 1.21 \pm 0.01$$

Per cui

$$\frac{\rho_B \lambda_T^3}{\rho_F \lambda_T^3} = \frac{2.60}{1.21} = \frac{\langle N \rangle}{V_1} \frac{V_2}{\langle N \rangle} = \frac{(L_x - \ell) A}{\ell A} = \frac{L_x}{\ell} - 1$$

$$\Rightarrow \quad \frac{\ell}{L_x} = \frac{1}{1 + \frac{2.60}{1.21}} = 0.318$$

ESERCIZIO 2, 16.12.22

① * $$g(\epsilon) = \frac{1}{\cancel{L^D}} \int \frac{d\bar{p}}{\left(\frac{h}{\cancel{L}}\right)^D} \delta\left(\epsilon - \epsilon_F\left(\frac{p}{p_F}\right)^\gamma\right)$$

$$= \frac{1}{h^D} \int_{\Omega_D} d\Omega_D \int_0^\infty dp\, p^{D-1} \delta\left(\epsilon - \epsilon_F\left(\frac{p}{p_F}\right)^\gamma\right) = \frac{\Omega_D}{h^D} \frac{p_E^{D-1}}{(\gamma \epsilon_F / p_F^\gamma)\, p_E^{\gamma-1}} \theta(\epsilon)$$

ove $E = \epsilon_F\left(\frac{p}{p_F}\right)^\gamma$ implica $p_E = \left(\frac{E}{\epsilon_F}\right)^{1/\gamma} p_F$

$$\Rightarrow \quad g(\epsilon) = \frac{\Omega_D}{h^D} \frac{p_F^\gamma}{\gamma \epsilon_F} p_E^{D-\gamma} \theta(\epsilon) = \frac{\Omega_D}{h^D} \frac{p_F^\gamma}{\gamma \epsilon_F} \left(\frac{E}{\epsilon_F}\right)^{\frac{D}{\gamma}-1} p_F^{D-\gamma} \cdot \theta(\epsilon)$$

$$\boxed{g(\epsilon) = \frac{\Omega_D}{h^D} \frac{p_F^D}{\gamma \epsilon_F} \left(\frac{E}{\epsilon_F}\right)^{D/\gamma - 1} \theta(\epsilon) \equiv c\, E^{D/\gamma - 1} \theta(\epsilon)}$$

②

$$\frac{1}{V} \ln[Z(V, T, z)] = \frac{g_s}{V} \sum_{\bar{p}} \ln\left[1 + z e^{-\beta\epsilon(s_z, \bar{p})}\right]$$

$$= g_s \int_0^\infty dE \ln\left[1 + z e^{-\beta E}\right] c\, E^{\frac{D}{\gamma}-1}$$

$$= g_s c \int_0^\infty dE \sum_{n=1}^\infty (-1)^{n-1} \frac{z^n e^{-\beta E n}}{n} E^{D/\gamma - 1} \qquad \boxed{s = \beta E n}$$

$$= c\, g_s \sum_{n=1}^\infty (-1)^{n-1} \frac{z^n}{n^{D/\gamma+1}} (k_B T)^{D/\gamma} \int_0^\infty ds\, s^{D/\gamma - 1} e^{-s}$$

$$\beta P = \frac{1}{V} \ln[Z(V,T,z)] = C g_s f_{\frac{D}{\gamma}+1}(z) (k_B T)^{D/\gamma} \Gamma\left(\frac{D}{\gamma}\right)$$

$$\boxed{\beta P = \frac{\Omega_D}{h^D} \frac{p_F^D}{\gamma} \frac{g_s}{\epsilon_F^{D/\gamma}} \Gamma\left(\frac{D}{\gamma}\right) (k_B T)^{D/\gamma} f_{\frac{D}{\gamma}+1}(z)}$$

(3)

$$n = z \frac{\partial}{\partial z} \frac{1}{V} \ln[Z(V,T,z)] \Big|_{V,T}$$

$$\Rightarrow \quad n = \frac{\Omega_D}{h^D} \frac{p_F^D}{\gamma} \frac{g_s}{\epsilon_F^{D/\gamma}} \Gamma\left(\frac{D}{\gamma}\right) (k_B T)^{D/\gamma} f_{D/\gamma}(z)$$

$$\tilde{C} = \frac{\Omega_D p_F^D g_s}{h^D \gamma \epsilon_F^{D/\gamma}} \Gamma\left(\frac{D}{\gamma}\right)$$

$$n = \tilde{C} (k_B T)^{D/\gamma} f_{D/\gamma}(z)$$

(4)

$$\frac{E}{V} = -\frac{\partial}{\partial \beta} \left[\frac{1}{V} \ln Z\right] \Big|_z = -\frac{\partial}{\partial \beta} \left[\tilde{C} \beta^{-D/\gamma} f_{D/\gamma+1}(z)\right] \Big|_z$$

$$= \tilde{C} \frac{D}{\gamma} \beta^{-\frac{D}{\gamma}-1} f_{D/\gamma+1}(z)$$

$$= \frac{D}{\gamma} \beta^{-1} \tilde{C} \beta^{-D/\gamma} f_{D/\gamma+1}(z)$$

$$= \frac{D}{\gamma} \beta^{-1} \beta P = \frac{D}{\gamma} P$$

$$\boxed{P = \frac{\gamma}{D} \frac{E}{V}}$$

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Ricordiamo che

$$\int dx\, \delta(a - f(x))\, g(x) = \sum_{i=1}^{p} \frac{g(x_i)}{|f'(x_i)|}$$

$$f(x_i) = a \qquad i = 1, \dots, p$$