Condensed Matter Physics II – Second intermediate exam
A.A. 2012-2013, June 21 2013

(time 3 hours)

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1 *Phonons in a linear chain with next-neighbour springs.*

Consider a lineae chain of atoms with mass M ands equilibrium spacing a. Each atom is coupled to the nearest neighbours by springs with elastic constant K_1 ed to the next neighnbours by springs with elastic constants K_2.

1. Write down the potential energy of the chain in terms of the deviation $u(n)$ of the atoms from the equilibrium position.
2. Obtain the equation of motion of the n-th atom.
3. Prove that $u(n) = \epsilon \exp[i(qna - \omega(q))t]$ is solution of the equation of motion and obtain $\omega^2(q)$
4. Demonstrate that for $q \to 0$ $\omega(q) \simeq cq$ and give the expression of c in terms of the physical parameters characterizing the problem.
5. Consider now (and only now!) $K_2 = K_1/2$ and demonstrate that for $0 < q \leq \pi/a$ $\omega^2(q) > 0$.
6. Demonstrate that $d\omega(q)/dq$ vanishes at a wavevector $0 < q < \pi/a$ and provide its value. Provide a qualitative sketch of $\omega(q), \quad 0 \leq q \leq \pi/a$.
   [You may use the results found in answering point 5) and $d\omega^2(q)/dq = 2\omega(q) \times d\omega(q)/dq$ to look for the zero of $d\omega^2(q)/dq$.]

   In case you need it: $cos(x) = -1/2$ at $x = 2\pi/3$ and $x = 4\pi/3$

   The questions have the following partial marks: per 2,2,4,2,3,5.

Exercise 2: *Superconductors*

Consider a metal that at kow temperature has a specific heat $c_v = \gamma T$, with $\gamma = 4.410^{-4} cal - mole^{-1} K^{-2}$ and a Debye temperature $\Theta = 170K$. We also know that the metal becomes superconductor at $T = 3.72K$. In the following assume a BCS superconductor,

1. Evaluate the electronic density of states per unit volume in $(cm^{-3} eV^{-1})$.
2. Evaluate ω_D in s^{-1}.
3. What's the value of V_0, the average attraction between electrons, in eV?
4. What's the value of the zero temperature gap, in eV?
5. Knowing that the effective electron mass in units of m_e is 1.32, evaluate the electron density in cm^{-1}.
6. Evaluate the correlation length in the superconducting phase a $T = 0$.