Condensed Matter Physics II. – A.A. 2018-2019, June 14, 2019

(*time 3 hours)*

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTA BENE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *One-dimensional linear chain with interactions beyond the nearest neighbours.*

Considerino the oscillations of a linear atomic chain, with interatomic distance a and mass M, when interactions beyond the nearest neighbours are present.

1. Write the potential energy of the chain with $u(n)$ the displacement from equilibrium position na of the n-th atom and $K_m = Ke^{-\mu|m|}$ the force constant of order m, $|m| \geq 1$.
2. Obtain the force acting on the l-th atom.
3. Obtain the condition under which a solution of the type $e^{i(qal-\omega t)}$ satisfies the equations of motions, i.e., obtain the dispersion relation for the squared frequency, $\omega(q)^2$.
4. Evaluate the infinite sum in $\omega(q)^2$, to obtain a explicit function of $e^{-\mu}$ and $[sin(qa/2)]^2$. You'll need to recall the formula for the sum of the geometric series.
5. Calculate the velocity of sound.
6. Provide a qualitative graphical comparison between the $\omega(q)$ found above for $e^{-\mu} = 1/2$ and that of a harmonic linear chain with only nearest neighbors springs of elastic constant K, i.e., with frequency $\omega_1(q) = 2\sqrt{K/M}\sin|qa/2|$. Exploit the values of the sound velocity and the values of the frequencies at $q = \pi/a$ in the two cases and answer (with detailed motivation) the question: is $\omega(q)^2$ a monotonic function for $0 < q < \pi/a$?

Exercise 2 : *BCS superconductor.*

Consider a metal which in the normal state has a specific heat at low temperature $c_v = \gamma T$, with $\gamma = 1.85 \times 10^3 \cdot erg \cdot cm^{-3} \cdot K^{-2}$ and a Debye temperature $\Theta = 275K$. The metal becomes superconductor at $T_c = 9.26K$ and the mass of the conductions electrons obeys: $m_c/m_e = 12$.

1. Calculate the electronic (energy) density of states at the Fermi energy(in $cm^{-3}eV^{-1}$). Use eq. 2.80 of the textbook.
2. What is the value of ω_D (the Debye frequency)?
3. What is the value of V_0 (in BCS theory) in eV; V_0 being the average attraction appearing in eq. 34.16 of the textbook?
4. Obtain the superconductor energy gap Δ in eV at $T = 0$?
5. Keeping in mind that the conduction electrons have an effective mass, obtain the value of the correlation length ξ_0 in the superconducting phase at $T = 0$.
6. Estimate London's Λ (in $\AA$)