Condensed Matter Physics II. – A.A. 2017-2018, june 1, 2018

(*time 3 hours*)

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *Magnetic specific heat*

Consider a 3D Heisenberg ferromagnet, in the absence of external magnetic fields. Assume that the spins have magnitude S and lie on a BCC lattice with lattice parameter (of the cubic cell) a. Assume only nearest-neighbours exchange interactions, with exchange coupling constant J. Assume that excited states are described by spin waves.

1. Consider the specific heat per unit volume, c_v^M , due to spin waves in the limit of low temperatures and show that it is possible (i) to approximate the spin wave energies $\epsilon(\mathbf{k})$ to leading order in k and (ii) extend the k-space integral over the FBZ to to the entire k-space.

   The problem is very similar to that of the low-temperature specific heat of lattice vibrations c_v^L: one considers spin waves (or magnons) in place of phonons.

2. Write down the spin wave energies $\epsilon(\mathbf{k})$ for small values of ka and to leading order in k, as function of S, a, J, k.
3. Obtain a closed expressione for c_v^M as function of K_B, T, S, J, n where n is the density. Recall that in the BCC there are 2 atoms per cubic cell. Also $\zeta(5/2) \simeq 1.34$.
4. Compare c_v^M with c_v^L as given in eq. (23.27) of the textbook and say which of the two specific heats becomes predominant at low temperatures.
5. Give the expression of the temperature T^* at which $c_v^M = c_v^L$.
6. Calculate the value of T^*, for $S = 1/2$, $J = 0.1eV$, $\Theta = 400^oK$, with Θ the Debye temperature.

Exercise 2: *Meissner effect*

Let's consider a superconductor occupying the region $0 \leq z \leq a$, otherwise infinite in x and y directions. In the region $z \leq 0$ there is a magnetic field $\mathbf{B}_1 = B_1 \hat{y}$ while in $z \geq a$ there is a magnetic field $\mathbf{B}_2 = B_2 \hat{y}$.

1. Let's assume that, given the simmetry of the problem, for $0 \leq z \leq a$ the magnetic field depends only on z, i.e., $\mathbf{B} = \mathbf{B}(z)$. Determine the form of the two indipendent solutions of the London equation, with Λ the London length.

2. Provide the boundary conditions in the superconducting region on $\mathbf{B}(z)$ at 0 and a, motivating them.

3. Use the above boundary conditions to determine the magnetic field profile within the superconductor.

4. Calculate the electric current in the superconductor region.

5. Provide the expression of the magnetization density $\mathbf{M}(z)$ inside the superconductor when $\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_0$.

6. Calculate the average magnetization density $\langle \mathbf{M} \rangle = (1/a) \int_0^a dz \mathbf{M}(z)$ in the superconductor; from this obtain the magnetic susceptibility when $\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_0$ for (i) $a \gg \Lambda$ and (ii) $a \ll \Lambda$.