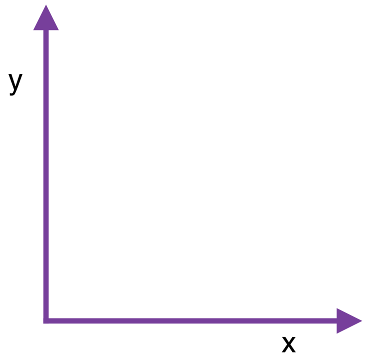
Condensed Matter Physics II. – A.A. 2014-2015, June 11 2015

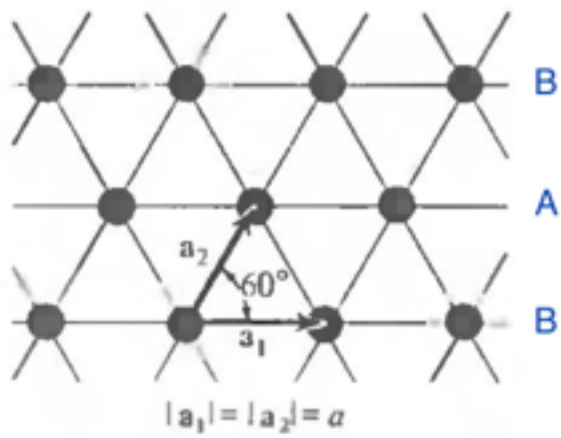
(time 3 hours)

Solve the following two exercises: (i) Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded; (ii) if you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: Frustated antiferromagnet: mean field

Consider N spins 1/2, in the presence of a magnetic field $\mathbf{H}=H\hat{y}$, at the sites of a triangular lattice (2 dimensions!) with nearest neighbours antiferromagnetic coupling $J=-K_BT_N$ and K_B the Boltzmann constant and T_N a temperature. The triangular lattice can be seen as a collection of one dimensional lattices (say in the x direction) with lattice parameter a, parallel to each other. The lattice has area A, so that the density of sites is n=N/A. Moreover that lattice can be seen as made by two sublattices, *A* and *B*, made by lines of type *A* and *B* respectively.





1. First of all consider independent spins (1/2) in the external magnetic field $\mathbf{H}=H\hat{y}$ and compute the free energy of a spin in the presence of the field $\mathbf{H}$.
2. Using the result of the previous point, compute the average magnetization $\mathcal{M}$ of a single spin.
3. Write down the hamiltonian of the interacting system.
4. Make a mean field approximations, assuming that the spins in the alternating horizontal *lines A* and *B* have average values $\langle S_x\rangle=0$ and $(n\mu_B g/2)\langle S_y\rangle = M_A$ on sublattice *A* and $\langle S_x\rangle=0$ and $(n\mu_B g/2)\langle S_y\rangle=M_B$ on sublattice *B*. Calculate the effective magnetic fields h_A and h_B, respectively at a site *A* and *B*. Here, M_A and M_B are sublattice magnetization densities: $M_A=(n/2)\,\mathcal{M}_A$, $M_B=(n/2)\,\mathcal{M}_B$, and $\mathcal{M}_A$ and $\mathcal{M}_B$ are single spin magnetizations, as calculated at point 2 above. NOTE: the effective field at a site is obtained adding to the external field the interaction with the 6 nearest neighbours, some of which are on sublattice *A* and some on sublattice *B*.
5. Write down the two selfconsistent equations for M_A and M_B when H=0. Remember that $\mathcal{M}_A$ and $\mathcal{M}_B$ are each function of the effective fields at the chosen sublattice.
6. Assuming that $M_A=-M_B$, show that there exists a critical temperature T_c, below which non zero sub lattice magnetizations are predicted.

Exercise 2: Lattice specific heat in D dimensions

Consider a lattice in dimension D, with *acoustic* phonon branches with small wave vector dispersion $\omega_s(\mathbf{k})=\omega_{0s}\,(k/q_s)^\nu$, s=1,2,.. D, $\nu>0$ and ω_0 and q_s a given frequency and a given wave vector. Denote with Ω_D the *solid angl*e in D dimensions.

1. Express the energy density (energy per unit volume) due to phonons at finite temperature in terms of an integral containing the phonon density of states $g(\omega)$ and the Plank distribution.

2. Demonstrate that as $T\to 0$ the energy density is determined solely by the density of states for $\omega\to 0$.

3. As the optical branches (if there is more than one atom in the basis) have finite energy for every k in the FBZ, at small energy (frequency) the phononic density of states $g(\omega)$ is completely determined by the acoustic branches at small k. Calculate $g(\omega)$ at small frequencies.

4. Use the density of states found above in the expression for the energy density found at point 1 and extend the frequency integral to infinity, motivating the accuracy of such a choice.

5. Manipulate the expression for the energy density found at the previous point in such a way to obtain an integral that is temperature independent, and discuss its convergence properties.

6. Obtain the dependence on temperature, as $T\to 0$, of the lattice specific heat.