Condensed Matter Physics II. – A.A. 2013-2014, June 8 2013

(time 3 hours)

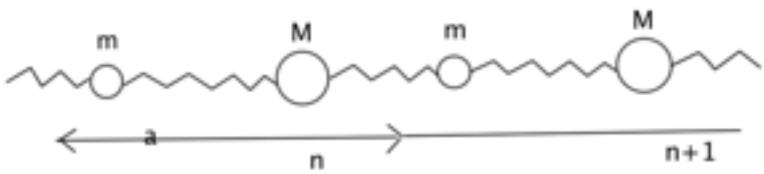
Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: Harmonic linear chain with two atoms per unit cell

Consider a linera harmonic chain with 2 atoms (with masses m and M) in a unit cell of length a. The atoms are connetced to the nearest neighbors by springs with harmonic constant K; specialize to $M \geq m$.



1. Write to total Potential energy of the linear chain when the atoms are displaced from the equilibrium positions: the displacement of atoms from equilibrium in the n-th cell will be denoted by $u_1(n)$ and $u_2(n)$, respectively for the atoms with mass m and M.

2. Write down the equation of motion form the atoms in the n-th cell.

3. Making the ansatz that the displacement have a wavelike behavior $u_1(n,t)=\varepsilon_1 \exp(i(qna - \omega t)$ and $u_2(n,t)=\varepsilon_2 \exp(i(qna - \omega t)$, obtain the allowed values of frequences $\omega(q)$, i.e. the dispersion of phonon branches.

4. Give the values of the obtained 2 phonon branches at q=0 and at $q=\pi/a$.

5. Observing that for q in $[0,\pi/a]$ one of the two branches is an increasing function of q and the other a decreasing function of q, give a qualitative sketch of the dispersion of the two branches.

6. Specialize the above sketch to the case in wich m=M and try to explain what happes in such limit.

Exercise 2: Magnetic specific heat

Consider a ferromagnet in 3 dimension, described by a Heseinberg hamiltonian and assume that the energies of low-lying excited states in the presence of a magnetic field H may be taken as

$$E(\{n_{\mathbf{k}}\}) = E_0 + g\mu_B H + \sum_{\mathbf{k}} n_{\mathbf{k}} \epsilon(\mathbf{k})$$

with
$$n_{\mathbf{k}} = 0, 1, 2, 3, ... \quad \text{and} \quad \epsilon(\mathbf{k}) = 2S \sum_{\mathbf{R}} J(\mathbf{R}) sin^2[\mathbf{K} \cdot \mathbf{R}/2]$$

where $\epsilon(\mathbf{k})$ is the energy of a spin wave with wavevector **k**.

1. Give (or derive, as you like) the expression for the thermal average $\langle n_{\mathbf{k}} \rangle$ at $H = 0$ at temperature T; here and in the following, consider H=0.

2. Write down the explicit expression of $\epsilon(\mathbf{k})$ for a BCC lattice, considering only nearest neighbor interactions and setting $\epsilon_0 = 2SJ(\sqrt{3}a/2)$, with a the cubic lattice parameter.

3. Write down the expression of the energy at small temperature T, making approximations similar to those employed for the lattice specific heat at small temperature, exploiting the fact that $\epsilon(\mathbf{k})$ tends to 0 as **k** tends to 0.

4. Obtain the T dependence of the enerrgy at small temperature.

5. Calculate the specific heat C_M at small temperature.

6. Consider now that the atoms in the crystal under examination also have small vibrations producing a lattice specific heat C_L. Say which of C_M and C_L dominates as $T \Rightarrow 0$.