Condensed Matter Physics II. – A.A. 2010-2011, June 10 2011

(time 3 hours)

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Esercizio 1 *Localized excitation kin a linear chain.*

Consider the harmonic vibrations (phonons) in an infinite linear chain of equispaced atoms, with lattice parameter a, and springs of constant G connecting each atom to its nearest neighbors.

1. (4 points) Write down the independent solutions $u_{\pm}(q, n, t)$ of the dynamical problem as plane waves, with q, n, t respectively a wavevector in the FBZ, a lattice position and time. You may conveniently think of a chain of length $L = Na$, with PBC. For each $-\pi/a \leq q < \pi/a$ $\pm$ denotes the mode respectively with $\pm\omega(q)$.
2. (6 points) Consider now a linear combination of the modes with amplitudes (i) $-a_+(q) = a_-(q) = a(q) = C\,exp(-|q|l)$ and $C = -\mathrm{i}\,(\pi/\mathrm{L})(\mathrm{l\,a}/10)$, assuming (ii) $l \gg a/\pi$, so that only acoustic modes have an appreaciable weight in the linear combination. Calculate

   $$u(n,t) = \sum_{\sigma=\pm,q} a_\sigma(q) u_\sigma(q,n,t).$$

   We remark that due to condition (ii) above, in the linear combination the mode dispersion can be taken acoustic and the integral over q can be extended to all q-space, i.e., over $[-\infty, \infty]$.
3. (2 points) Are there atoms displaced from the equilibrium positions at $t = 0$.
4. (3 points) Calculate the speed of each atom at $t = 0$.
5. (3 points) Which atoms are displaced from equilibrium at $t = ma/c$, with $ma \gg l$ and c the sound velocity: please, answer by giving a qualitative sketch of the displacements along the chain!

   Note: in the version given in class the definition of C did not include the factor $-\mathrm{i}$, with i *the imaginary unit*!

Esercizio 2 *Pauli susceptibility*

Consider a non interacting electron gas in 3 dimensions, in a uniform magnetic field $\mathbf{B} = B\hat{z}$.

1. (3 points) Write the kinetic energy $T_\uparrow$ for the spin up electrons, assuming that their number is $N_\uparrow$; similarly write $T_\downarrow$ for the spin down electrons, if their number is $N_\downarrow$.
2. (3 points) Write down the energy E_z giving the interaction of the electron spins with the magnetic field $\mathbf{B}$.
3. (3 points) Use $N_\uparrow = (N/2)(1+\zeta)$, $N_\downarrow = (N/2)(1-\zeta)$ to express the total energy $E = T_\uparrow + T_\downarrow + E_z$ in terms of N, ζ and B, namely to get $E(N, \zeta, B)$.
4. (3 points) Minimize $E(N, \zeta, B)$ with respect to ζ, i.e. imposes the extrememum condition, at given N and B and write the resulting relation between ζ a B.
5. (3 points) Solve the above equation in the limit of small ζ.
6. (3 points) Calculate Pauli susceptibility $\chi_P = -\mu_B d\zeta/dB$. Say what is the relation between the present result and the one in the A.M. book.