COMPITO DEL 19/11/14

ESERCIZIO 1: Trappola armonica bidimensionale

① $Q_N(S,T) = \frac{q^N}{N!}$, $q = \frac{1}{\lambda^2}\int_S d\bar{r}\, e^{-\beta \frac{m\omega^2 r^2}{2}} = \frac{2\pi}{\lambda^2}\int_0^R dr\, r\, e^{-\frac{r^2}{\ell^2}}$

$\ell = \sqrt{\frac{2 K_B T}{m\omega^2}}$

$q = \frac{2\pi\ell^2}{\lambda^2}\int_0^{R/\ell} dy\, y\, e^{-y^2} = \frac{\pi\ell^2}{\lambda^2}\int_0^{(R/\ell)^2} dt\, e^{-t} = \frac{\pi\ell^2}{\lambda^2}\left(-e^{-t}\right)\Big|_0^{(R/\ell)^2} = \frac{\pi\ell^2}{\lambda^2}\left(1-e^{-R^2/\ell^2}\right)$

$Q_N(S,T) = \frac{1}{N!}\left[\frac{\pi\ell^2}{\lambda^2}\left(1-e^{-R^2/\ell^2}\right)\right]^N \simeq \left[\frac{e}{N}\pi\frac{\ell^2}{\lambda^2}\left(1-e^{-R^2/\ell^2}\right)\right]^N$

$A(N,S,T) = -K_B T N \ln\left[\frac{e}{N}\pi\frac{\ell^2}{\lambda^2}\left(1-e^{-R^2/\ell^2}\right)\right]$

② $P = -\frac{\partial A}{\partial S} = -\frac{1}{2\pi R}\frac{\partial A}{\partial R} = \frac{K_B T N}{2\pi R}\frac{(2R/\ell^2)\, e^{-R^2/\ell^2}}{1-e^{-R^2/\ell^2}} = \frac{K_B T N}{\pi\ell^2}\frac{e^{-R^2/\ell^2}}{1-e^{-R^2/\ell^2}}$

③ $\mu = \frac{\partial A}{\partial N} = K_B T \ln\left[\frac{N\lambda^2}{\pi e \ell^2}\frac{1}{1-e^{-R^2/\ell^2}}\right] + K_B T = K_B T \ln\left[\frac{N\lambda^2}{\pi\ell^2}\frac{1}{1-e^{-R^2/\ell^2}}\right]$

④ $\rho(r) = N\langle\delta(\bar{r}-\bar{r}_1)\rangle = N\frac{\int d\bar{r}_1\, e^{-r_1^2/\ell^2}\delta(\bar{r}-r_1)}{\int d\bar{r}_1\, e^{-r_1^2/\ell^2}} = N\frac{e^{-r^2/\ell^2}}{\pi\ell^2\left(1-e^{-R^2/\ell^2}\right)}$

$P = K_B T \rho(R)$, $\mu = K_B T \ln\left[\lambda^2\rho(0)\right]$

EXERCISE 2

① $G(N, P, T)$

$$G = \frac{\partial G}{\partial N} N = \mu N$$

$$dG = \mu dN + V dP - S dT = d(\mu N) = N d\mu + \mu dN$$

$$\Rightarrow d\mu = (V/N) dP - (S/N) dT = v dP - s dT$$

$$\Rightarrow \boxed{\mu(P, T)}$$

② $F(N, V, T)$

$$F = \frac{\partial F}{\partial N} N + \frac{\partial F}{\partial V} V = \mu N - PV = F; \quad f = \frac{F}{N} = \mu - Pv$$

$$df = d\mu - P dv - v dP = v dP - s dT - P dv - v dP = -s dT - P dv$$

$$\Rightarrow \boxed{f(T, v)}$$

④ $H(N,P,S)$

$$H = \frac{\partial H}{\partial N} N + \frac{\partial H}{\partial S} S = \mu N + T S$$

$$h = H/N = \mu + Ts$$

$$dh = -s\cancel{dT} + Vdp + T d\lambda + \lambda\cancel{dT} = v dp + T d\lambda$$

$\Longrightarrow$ $\boxed{h(p,\lambda)}$

③ $E(N,V,S)$

$$E = \frac{\partial E}{\partial N} N + \frac{\partial E}{\partial V} V + \frac{\partial E}{\partial S} S = \mu N - pV + TS$$

$$e = E/N = \mu - pv + Ts = f + Ts$$

$$de = -s\cancel{dT} - p dv + \lambda\cancel{dT} + T d\lambda = -p dv + T d\lambda$$

$\Longrightarrow$ $\boxed{e(v,\lambda)}$