COMPITO 05.05.2017

ESERCIZIO 1

① $$E_I[\rho] = \frac{A}{3}\int dx\, \rho^3(x) - \frac{1}{2}\int dx\, \rho^2(x)$$

② $$E[\rho] = \frac{A}{3}\int dx\, \rho^3(x) - \frac{1}{2}\int dx\, \rho^2(x) + \int dx\, \rho(x) v(x)$$

$$\frac{\delta E}{\delta \rho(x)} = \mu = A\rho^2(x) - \rho(x) + v(x)$$

③ $\Rightarrow$ $$\rho(x) = \frac{1}{2A}\left[1 \pm \sqrt{1 - 4A(v(x) - \mu)}\right]$$

④ $$\frac{\delta^2 E}{\delta\rho(x)\delta\rho(x')} \;?\; \frac{\delta^2 E}{\delta\rho(x)\,\delta\rho(x')} = \frac{\delta}{\delta\rho(x')}\,\frac{\delta E}{\delta\rho(x)}$$

$$\Delta_1\left[A\rho^2(x) - \rho(x) + v(x)\right]$$

$$= 2A\rho(x)\,\delta\rho(x) - \delta\rho(x)$$

$$= \int dy\, \delta(x-y)\left[2A\rho(y) - 1\right]\delta\rho(y)$$

$$\frac{\delta}{\delta\rho(x')}\frac{\delta E}{\delta\rho(x)} = \left[2A\rho(x') - 1\right]\delta(x - x')$$

$$\Rightarrow \quad 2A\rho(x) - 1 > 0$$

$$\Rightarrow \quad 1 \pm \sqrt{1 - 4A(\sigma(x) - \mu)} - 1 > 0$$

Solo il segno + va bene

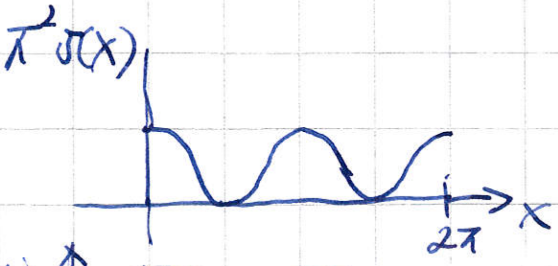
$$\begin{cases} \rho(x) = \frac{1}{2A}\left[1 + \sqrt{1 - 4A(\sigma(x) - \mu)}\right] \\ 4A(\sigma(x) - \mu) \leq 1 \end{cases}$$

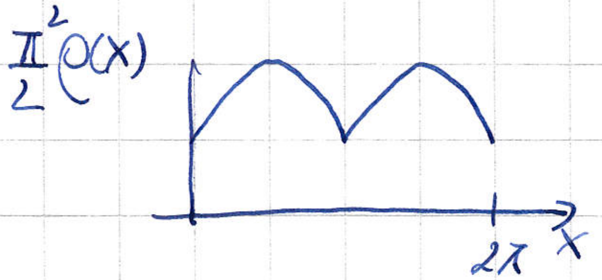
$$\begin{cases} \rho(x) = 0 \\ 4A(\sigma(x) - \mu) > 1 \end{cases}$$

(5) $$\rho(x) = \frac{2}{\pi^2}\left[1 + \sqrt{1 - 1 + \sin^2(x) + c^2}\right]$$

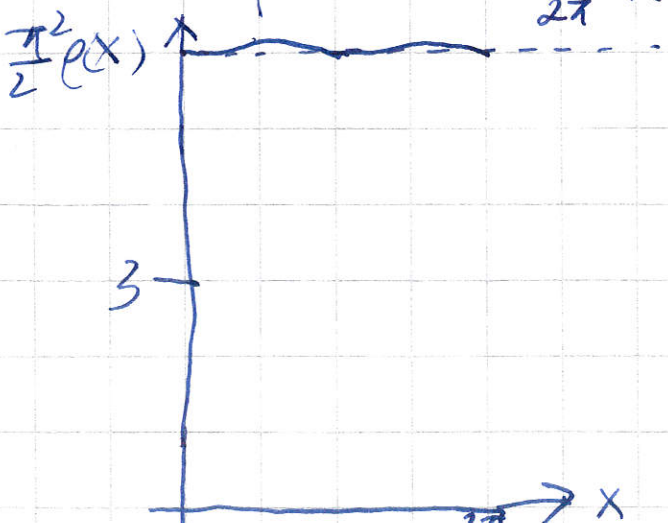
$$\rho(x) = \frac{2}{\pi^2}\left[1 + \sqrt{\sin^2(x) + c^2}\right] \quad \forall x$$

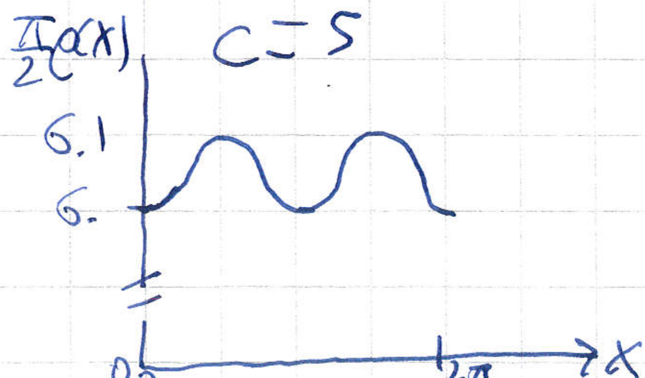
(6)





$c = 0$





$c = 5$

ESERCIZIO 2.

1 $$C = \frac{\sqrt{2}\sqrt{\det(M)}}{\hbar^3 \pi^2} \Rightarrow \frac{\det(M)}{m_e^3} = \frac{\pi^4 \hbar^6 C^2}{2\, m_e^3}$$

$$\frac{\det(M)}{m_e^3} = \frac{(3.14)^4 \cdot (1.05)^6 \cdot 10^{-162} \cdot (1.01)^2 \cdot 10^{78}}{2 \cdot (9.11)^3 \cdot 10^{-84}}$$

$$= 8.79 \cdot 10^{-2}$$

2 $$m_H(\vartheta,\varphi) = \sqrt{\frac{\det(M)}{\sin^2\vartheta\,(m_1\cos^2\varphi + m_2\sin^2\varphi) + m_3\cos^2\vartheta}}$$

$m_H(\vartheta,\varphi)$ does not depend on φ !

$$m_H(\vartheta,\varphi) = \frac{\sqrt{\det(M)}}{\sqrt{\sin^2\vartheta\,[(m_1 - m_2)\cos^2\varphi + m_2] + \cos^2\vartheta\, m_3}}$$

Independence on φ "necessarily" requires

$\boxed{m_1 = m_2 = m_\pi}$ at most 2 different eigenvalues

3 In units of m_e, $m_3 \equiv m_L$, $\sin\vartheta = 1$

$$m^*(H) = \frac{\sqrt{m_\pi^2\, m_L}}{\sqrt{m_\pi}} = \frac{\sqrt{8.79 \cdot 10^{-2}}}{\sqrt{m_\pi}} = 0.469$$

$$m_\pi = \frac{8.79 \cdot 10^{-2}}{(0.469)^2} = 0.400$$

$$m_L = \frac{\det(M)}{m_T^2} = \frac{8.78 \cdot 10^{-2}}{(0.400)^2} = 0.549$$

(4) $\bar{\beta} \equiv \hat{H} = (0, 0, 1)$

$$m_H^* = \sqrt{\frac{m_T^2\, m_L}{m_L}} = m_T$$

$$\bar{v}_\alpha = \bar{\gamma}_\alpha\, e^{\frac{eH}{c}\delta t} \qquad \delta = 0,\ \pm \frac{i}{m_T}$$

$$\begin{pmatrix} m_T\delta & -1 & 0 \\ 1 & m_T\delta & 0 \\ 0 & 0 & m_L\delta \end{pmatrix} \begin{pmatrix} \gamma_{\alpha,1} \\ \gamma_{\alpha,2} \\ \gamma_{\alpha,3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\delta = 0$, $\bar{\gamma}_0 = (0, 0, 1)$

$\delta = \pm \frac{i}{m_T}$, $\bar{\gamma}_\pm = (1, \pm i, 0)$

Le orbite nel piano ortogonale a (0,0,1) sono cerchi, quindi si ha una spirale lungo (0,0,1) con sezione trasversale circolare.

5 $\bar{\beta} = (1, 0, 0)$

$$m_H^* = \sqrt{\frac{m_T^2\, m_L}{m_T}} = \sqrt{m_T\, m_L}$$

$$\begin{pmatrix} m_T\delta & 0 & 0 \\ 0 & m_T\delta & -1 \\ 0 & 1 & m_L\delta \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\delta = 0 \qquad \bar{\gamma}_0 = (1, 0, 0)$$

$$\delta = \pm \frac{i}{\sqrt{m_T\, m_L}} \qquad \bar{\gamma}_\pm = \left(0, 1 \pm i\sqrt{\frac{m_T}{m_L}}\right)$$

Le orbite nel piano ortogonale a (1, 0, 0) sono ellissi, quindi si ha una spirale con sezione trasversa ellittica.

6 $$\omega_c = \frac{eH}{m_H c}$$

$$\omega_c = \frac{4.8 \cdot 10^{-10}\; H(g)}{0.430 \cdot 9.11 \cdot 10^{-28} \cdot 3 \cdot 10^{10}}$$

$$\omega_c = 4.08 \cdot 10^7\, s^{-1}\; H(g)$$

$$m_H = \sqrt{\frac{m_T^2\, m_L}{\frac{1}{2}(m_T + m_L)}} = \sqrt{\frac{(0.4)^2\, 0.549}{0.5(0.4 + 0.549)}} \cdot m_e = 0.430\, m_e$$