Condensed Matter Physics II. – A.A. 2021-2022, April 29, 2022

(time 3 hours)

Solve the following two exercises.

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *LDA (with exchange) for a 2D electron gas with linear dispersion*

Consider Electrons in 2D with a kinetic energy per electron linear in the wavevector k and the familiar pair interaction e^2/r. In plane-wave Hartree-Fock the kinetic energy per particle is $t(\rho) = A\sqrt{\rho}$, with $A = \alpha\sqrt{2\pi}/3$ and α a positive constant, and the exchange energy $\epsilon_x(\rho) = -B\sqrt{\rho}$ with $B = (4e^2\sqrt{2\pi})/(3\pi)$. Thus the energy per particle is

$$\epsilon(\rho) = (A - B)\sqrt{\rho} \equiv 2C\sqrt{\rho}/3.$$

Note that, depending on the values of A and B, C may be positive, zero o negative. $\rho = N/A$ is the Fermions density.

1. Write in LDA the internal energy functional $E_I[\rho]$ (kinetic + exchange energies only).
2. Using the result above, write the full energy functional $E[\rho]$ (kinetic + exchange energies + interaction with external potential) and impose the extremum condition on $F[\rho] = E[\rho] - \mu \int d\mathbf{r}\rho(\mathbf{r})$ (zero first functional derivative or first variation $\Delta_1 F[\rho]$.
3. Use the extremum condition to obtain an expression for $\rho(\mathbf{r})$.
4. Calculate the second variation $\Delta_2 F[\rho]$ and imposing its positivity obtain the condition under which the extremum is a minimum, yielding an equilibrium density $\rho(\mathbf{r})$.
5. Give the expression of the equilibrium $\rho(\mathbf{r})$ when

$$v(\mathbf{r}) = (v_0/2)\cos(2\pi x/l), \quad \mu = v_0/2,$$

   with v_0 and l positive.
6. Provide qualitative skecthes of v($\mathbf{r}$) and of $\rho(\mathbf{r})$.

Exercise 2: *Effective masses in a semiconductor*

Let's recall eq. (28.8) of AS:

$$m^*(\beta) = \sqrt{\frac{m_1 m_2 m_3}{m_1\beta_1^2 + m_2\beta_2^2 + m_3\beta_3^2}}, \qquad \beta_1^2 + \beta_2^2 + \beta_3^2 = 1$$

1. Performing experiments in a magnetic field one finds that varying β_1 or β_2 at fixed β_3 m^* does not change. What is the implication of this fact on m_1 and m_2. Motivate in detail your conclusions.
2. An experiment with $\beta_3 = 1$ yields $m*/m_e = 0.190$. This, together with the conclusion found in the previous point, provides the value of 2 of the eigenvalues of the mass tensor. Give them with 3 significant figures.
3. An experiment with $\beta_3 = 0$ yields $m*/m_e = 0.418$. This provides the third eigenvalue of the mass tensor. Give it with 3 significant figures.
4. The knowledge of the 3 eigenvalues of the mass tensor allows to evaluate the constant C multiplying $\sqrt{|E - E_{c,v}|}$ in eq. 28.14 of AM. Calculate C in $cm^{-3}erg^{-3/2}$
5. What is the shape of the orbits for a magnetic field lying along the $\hat{x}_2$ axis? Give a detailed derivation.
6. What will be the measured cyclotron frequency (in cm^{-1}) for a magnetic field with $\hat{H} = (1/\sqrt{3})(1, 1, 1)$ and H=1 gauss?