Condensed Matter Physics II. – A.A. 2011-2012, May 11 2012

(*time 3 hours)*

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *LDA for Fermions in a trap*

1. Give the energy per particle on non iteracting unpolarized Fermions of mass m in 3D in terms of the number density $n = N/V$.
2. Consider now a system of non interacting Fermions in an external potential $v(\mathbf{r})$ and write the total energy (kinetic + interaction with the external potential) resorting to the Local Density Approximation (LDA) for the kinetic energy.
3. Obtain an expression for the equilibrium one-body density (from the minimum energy principle).
4. Specialize now to the case in which $v(\mathbf{r}) = -V_0$ for $r \leq R$ and 0 elsewhere, with $V_0 > 0$, and give a qualitative plot of $n(r)$ when $-V_0 < \mu \leq 0$.
5. The same as above but for $\mu > 0$.
6. In which of the two cases above does the one-body density integrate to a finite number and why?

Exercise 2: *1D semiconductor*

Consider a one-dimensional semiconductor with 1 atom per cell (the cell has length a) and 2 electrons per atom. We shall just consider the 2 lowest energy bands, with dispersion: $E_1(k) = -\Delta + (\gamma/2)[-1 + cos(ka)]$ e $E_2(k) = \Delta - (\gamma/2)[-1 + cos(ka)]$ with γ, $\Delta > 0$.

1. Plot the two bands in the FBZ, indicating the values of minimum and maximum of each band.
2. A $T = 0$ is the system a metal or an insulator and why? Where it is likely to fall $\mu(T = 0)$?
3. Calculate the density of states in energy per unit length $g_1(E)$, relative to the band $E_1(k)$ and plot it; give a simpler expression of $g_1(E)$ valid near the maximum of $E_1(k)$.
4. Calculate the density of states in energy per unit length $g_2(E)$, relative to the band $E_2(k)$ and plot it; give a simpler expression of $g_2(E)$ valid near the minimum of $E_2(k)$.
5. Calculate the concentration of minority carriers in the two bands, $n_e(T)$ and $p_v(T)$, in the non degenerate regime at low temperature.
6. Imposing the equality $n_e(T)$=$p_v(T)$, valid for an intrinsic system, obtain $\mu(T)$: try and comment on the relation between your expression of $\mu(T)$ and the one in the textbook for 3 dimensions.