Condensed Matter Physics II. – A.A. 2020-2021, June 14 2021

(*time 3 hours)*

Solve the following two exercises, each has a maximum score of 18 for a total of 36. A score between 33 e 36 corresponds to 30 cum laude, between 30 e 32 is renormalized to 30 (the maximum official score, without laude).

NOTE:

- Give all details which help in understanding the proposed solution. Answers which only contain the final result or not enough detail will be judged insufficient and discarded;
- If you are requested to give evaluation/estimates, do so using 3 significant figures.

Exercise 1: *Velocit of sound in Metallic Na*

A crude model of the small oscillations of Na^+ in metallic sodium is easily obtained by considering the electron as forming a rigido homogeneous charge background. The ions are immagined to be in stabile equilibrium when at the lattice points. If one ion is displaced a small distance r from its equilibrium position, the restoring force is largely due to the electronic charge within the sphere of radius r centered at the equilibrium position. In the following write the number density of ions (or electron) as $\rho = (4\pi R^3/3)^{-1}$.

1. Calculate the force acting on an ion for a small displacement from its equilibrium position.
2. Calculate the frequency of a single ion set in oscillation. We shall denote such a frequency as Ω_{IS} (IS=Ion Sphere).
3. Evaluate Ω_{IS} numerically.
4. For large wavelength one may describe the motion of ions as an ionic plasmon. One obtains the frequency Ω_p of such plasmon just by substituting the electron mass with the ion mass in the espression providing the electron plasmon frequency ω_p. What is the relations between Ω_{IS} and Ω_p?
5. One way to take into account the response of electrons to the ions is to replace in either Ω_{IS}^2 or Ω_p^2 the coupling e^2 with $e^2/\epsilon(k)$ where $\epsilon(k)$ is the dielectric function of the electron gas at small wavevectors k. This amounts to assume that the ionic interaction is screened by electron. Show that this changes the plasma frequency into and acoustic mode.
6. Calculate the expression of the velocity of sound of such an acoustic mode in the case in which $\epsilon(k)$ is taken from eqs. (17.51) and (17.55) and give also a numerical estimate. Is the obtained result sound?

Exercise 2: Antiferromagnet on a hexagonal lattice

Consider a hexagonal lattice with sites occupied by spin 1/2. The properties of the system can be described in terms of a Heisenberg Hamiltonian with nearest-neighbour interactions with $J = -|J|$. We recall that a possible choice for primitive vectors and basis vectors are:

$$\mathbf{a}_1 = a(\sqrt{3}/2, 1/2), \quad \mathbf{a}_2 = a(\sqrt{3}/2, -1/2), \quad \mathbf{b}_1 = (0,0), \quad \mathbf{b}_2 = a(1/\sqrt{3}, 0).$$

1. Write the Hamiltonian H when a magnetic field h is present.
2. First consider the spin independent case $J = 0$ and obtain the expression of magnetization.
3. Consider now $J < 0$ and the possibility that the two triangular lattices (A and B), which form the hexagonal lattice, have different average magnetizations M_A and M_B. Making a mean field approximation obtain the effective fields h_A and h_B at the sites of the two triangular lattices.
4. Write down the average magnetizations M_A and M_B in terms of the effective fields, namely in terms of h, M_A and M_B.
5. Study the case in wich $h = 0$ to determine if a critical temperature T_c exists, such that for $T < T_c$ a spontaneous order exists with $M_A \neq 0$ and $M_B \neq 0$, but $M_A \neq M_B$.
6. Calculate the magnetic susceptibility for $T \gtrsim T_c$: how does it compare with the one found for an Heisenberg ferromagnet?