Short Time Fourier Transform (STFT)

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Fourier Transform

- Fourier Transform reveals which frequency components are present in a function:

$$f(x) = \sum_{u=0}^{N-1} F(u) e^{\frac{j2\pi ux}{N}}, \quad x = 0, 1, \ldots, N-1$$

(inverse DFT)

where:

$$F(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) e^{\frac{-j2\pi ux}{N}}, \quad u = 0, 1, \ldots, N-1$$

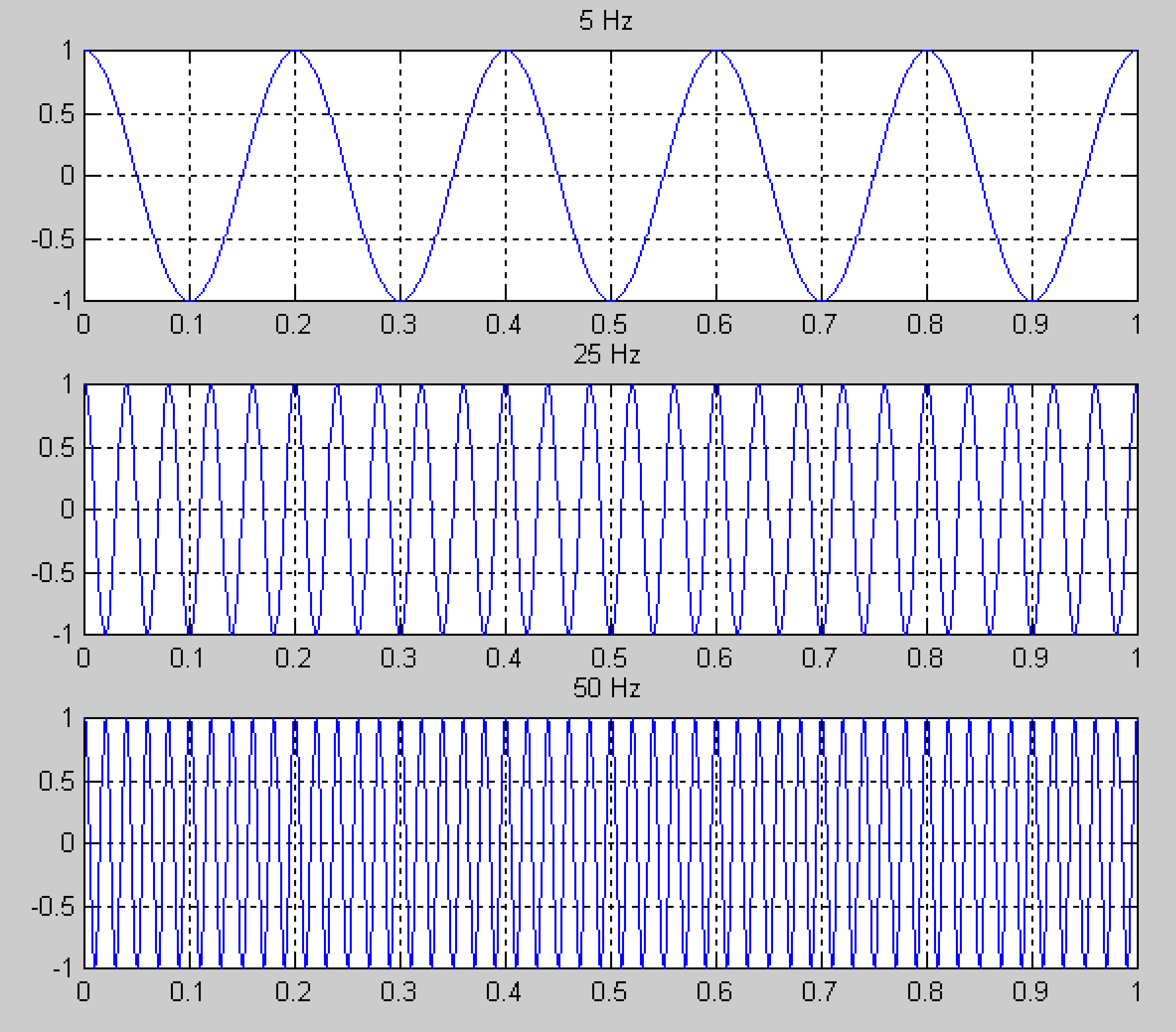
(forward DFT)

Examples

$$f_1(t) = \cos(2\pi \cdot 5 \cdot t)$$

$$f_2(t) = \cos(2\pi \cdot 25 \cdot t)$$

$$f_3(t) = \cos(2\pi \cdot 50 \cdot t)$$

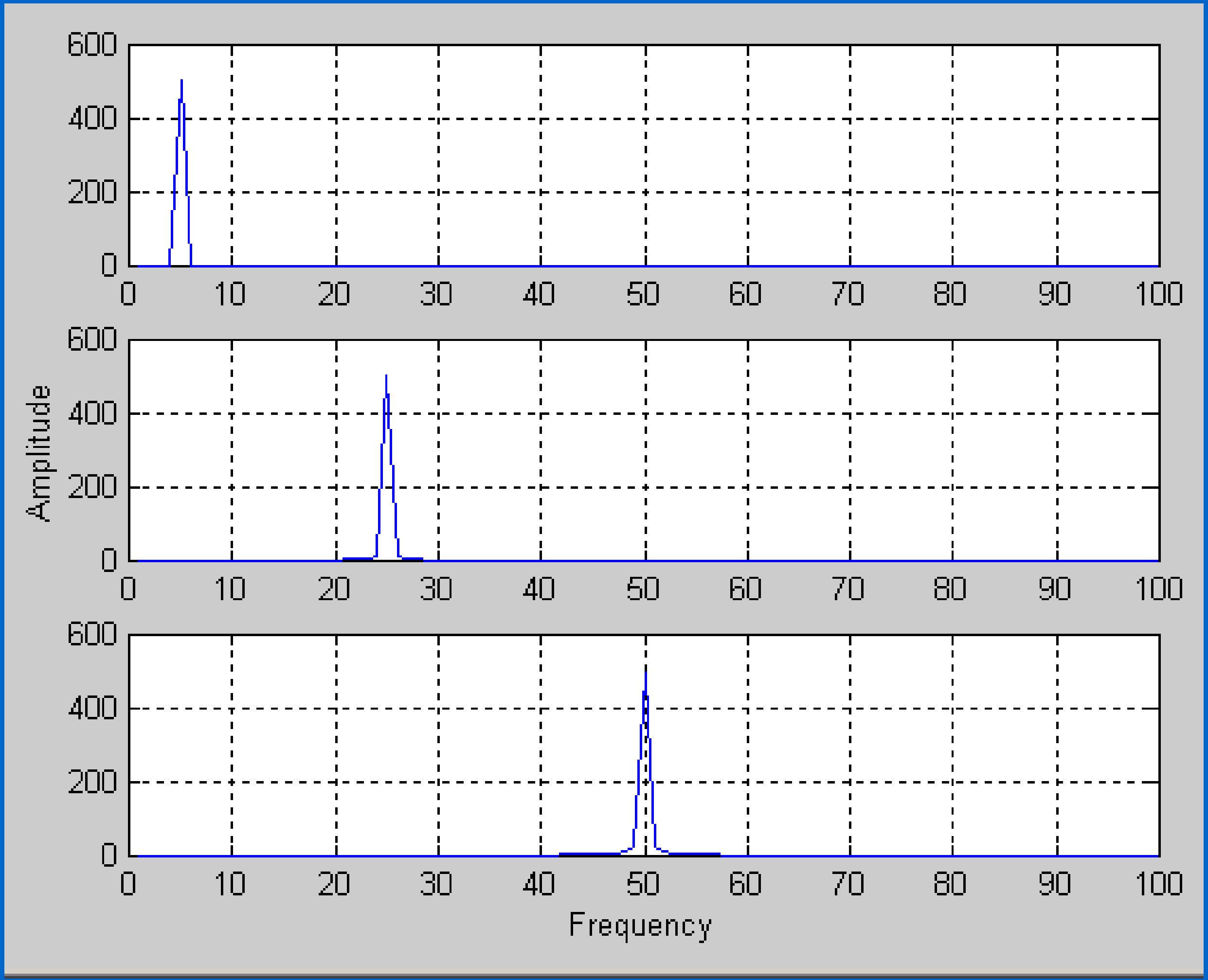


The plots below indicate that the continuous signals have been sampled at Fs = ... Hz (or possibly larger)

$F_1(u)$

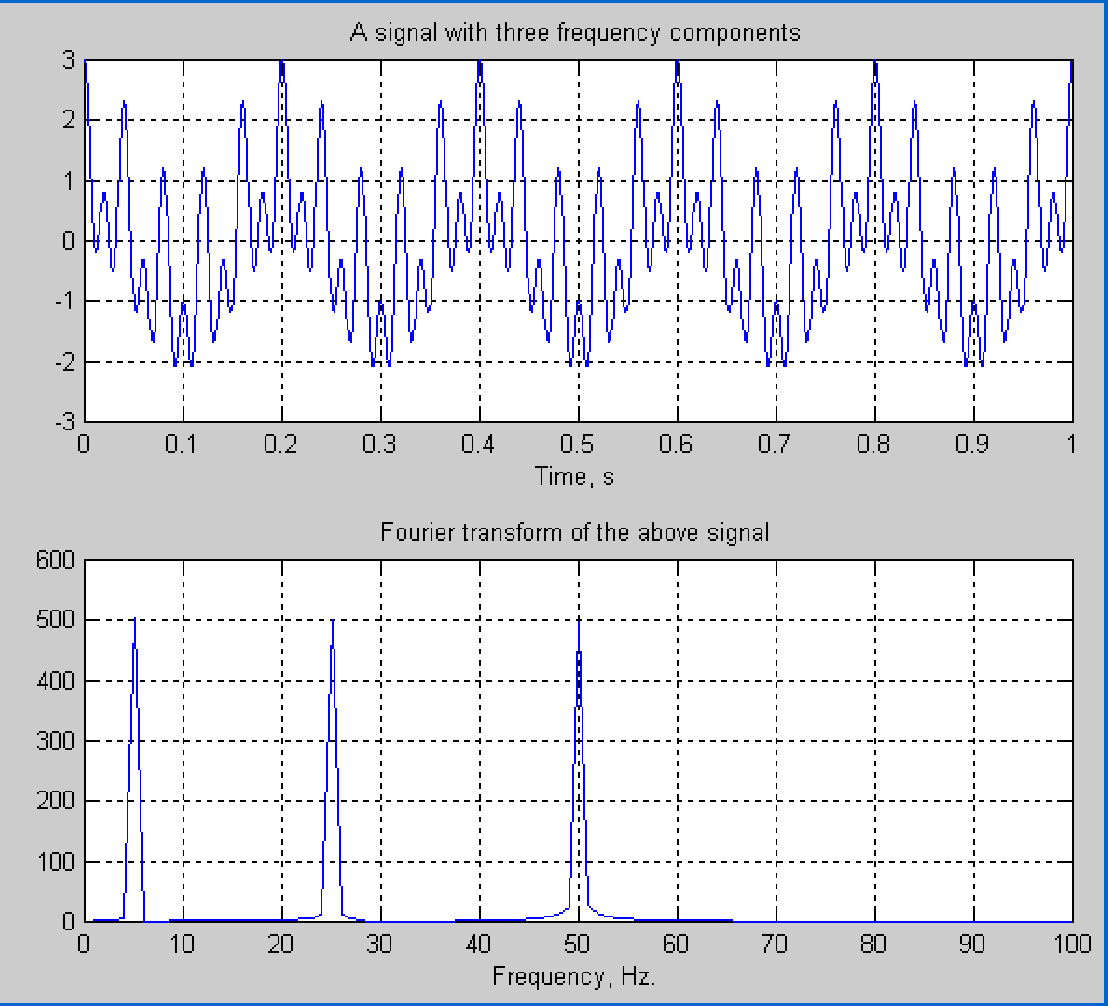
$F_2(u)$

$F_3(u)$



Fourier Analysis – Examples (cont'd)

$$f_4(t) = \cos(2\pi \cdot 5 \cdot t) + \cos(2\pi \cdot 25 \cdot t) + \cos(2\pi \cdot 50 \cdot t)$$



$F_4(u)$

Limitations of Fourier Transform

1. Cannot provide simultaneous time and frequency localization.

Limitations of Fourier Transform (cont'd)

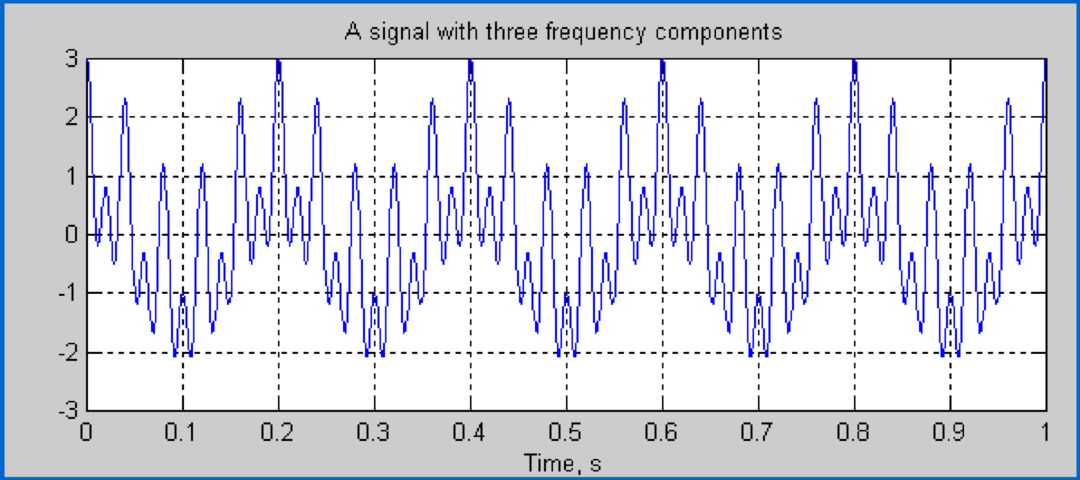
1. Cannot provide simultaneous time and frequency localization.

2. Not very useful for analyzing time-variant, non-stationary signals.

Stationary vs non-stationary signals

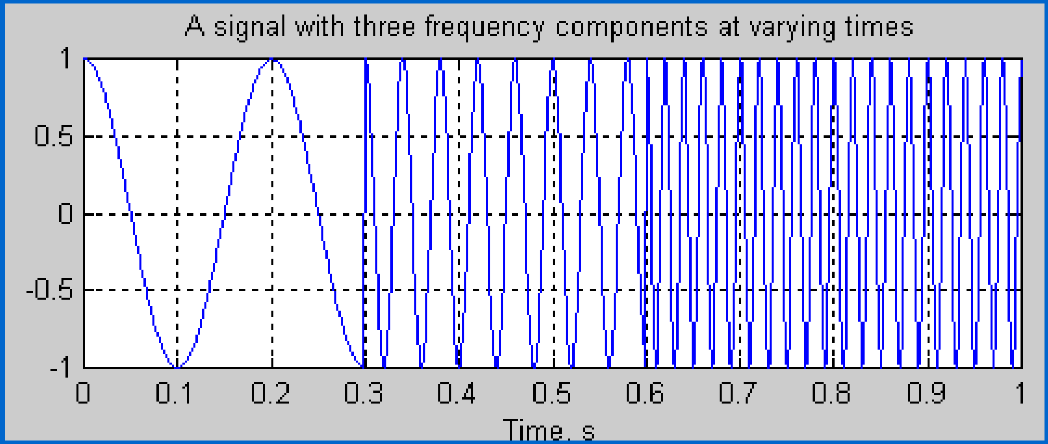
- Stationary signals: time-invariant spectra

$f_4(t)$



- Non-stationary signals: time-varying spectra

$f_5(t)$

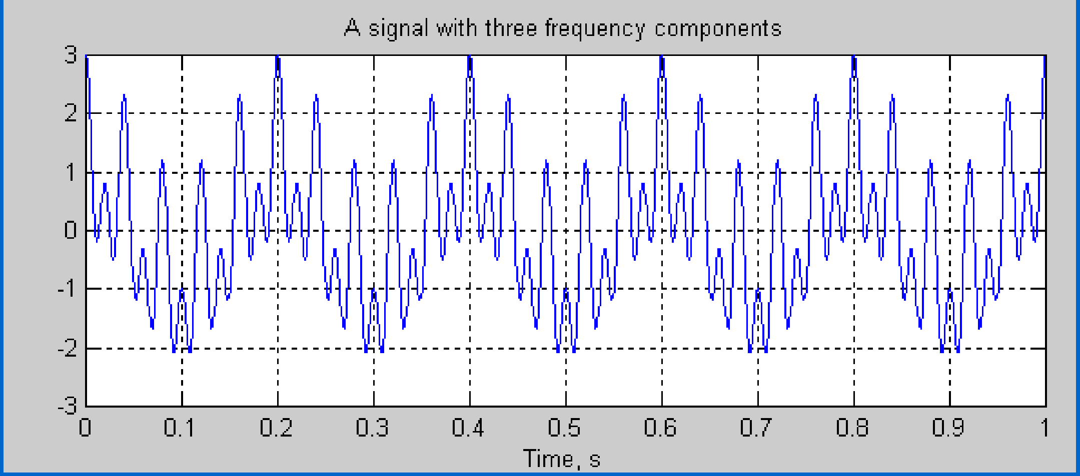


Stationary vs non-stationary signals (cont'd)

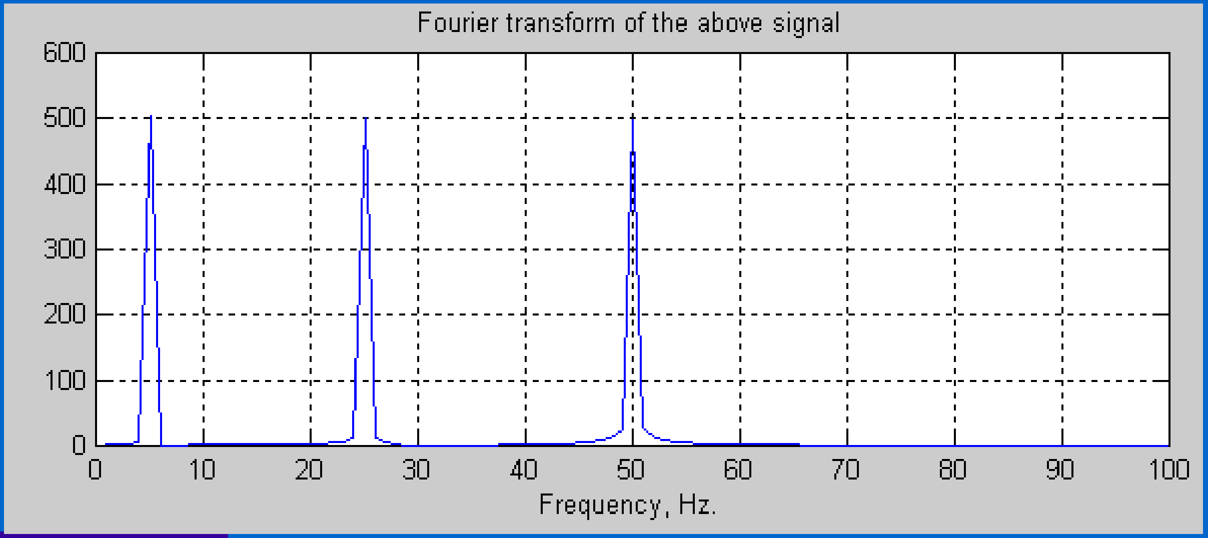
Stationary signal:

Three frequency components, present at all times!

$f_4(t)$



$F_4(u)$



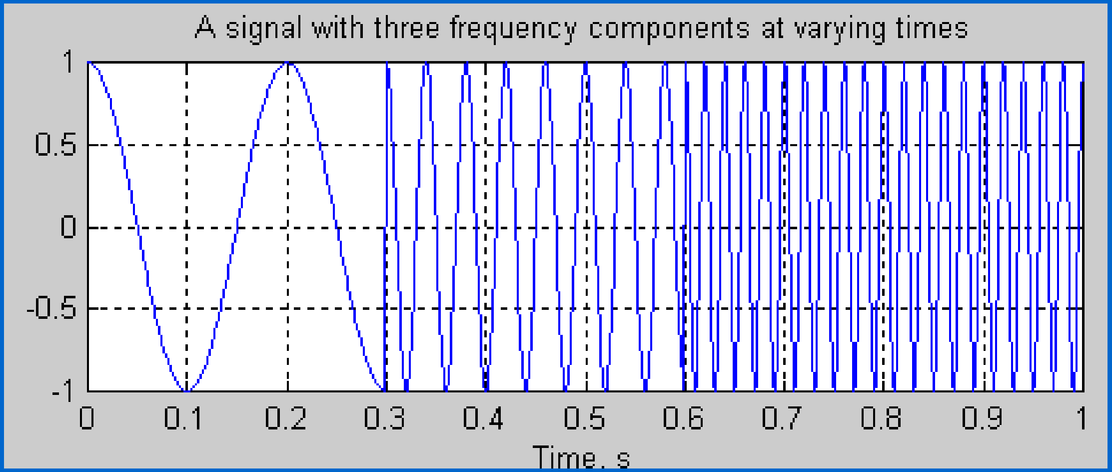
Stationary vs non-stationary signals (cont'd)

Non-stationary signal:

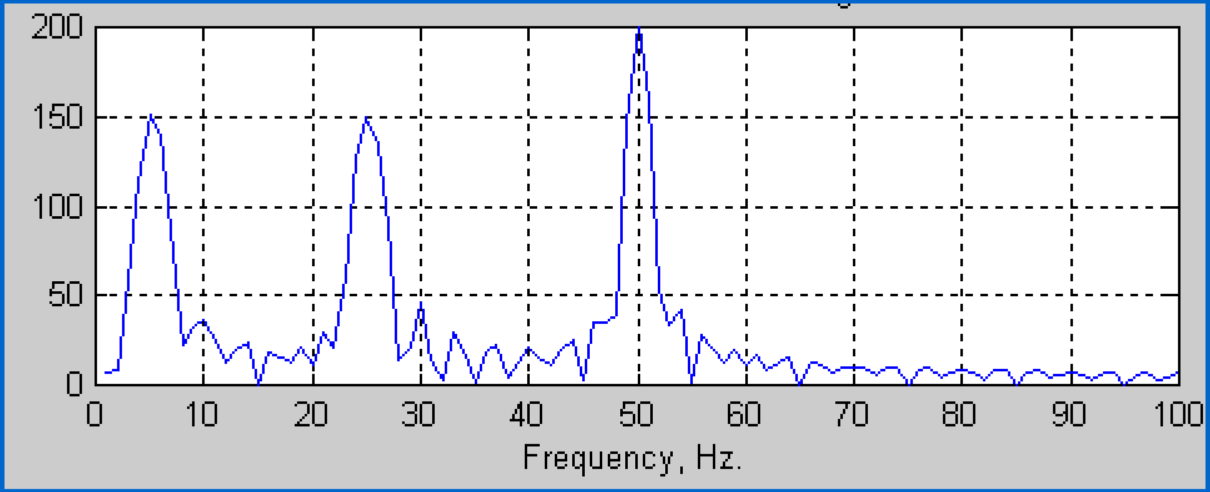
Three frequency components, NOT present at all times!

(note: *more freq. components are present due to signal transitions*)

$f_5(t)$

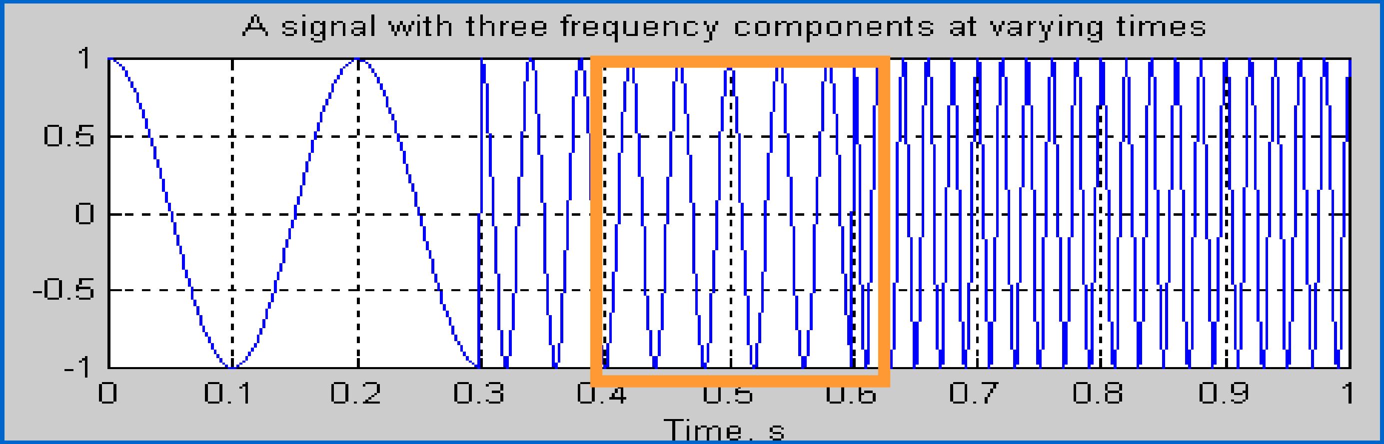


$F_5(u)$



Short Time Fourier Transform (STFT)

- Segment the signal into narrow time intervals (i.e., narrow enough to be considered stationary) and take the FT of each segment.
- Each FT provides the spectral information of a separate time-slice of the signal, providing simultaneous time and frequency information.



STFT - Steps

(1) Choose a window function of finite length

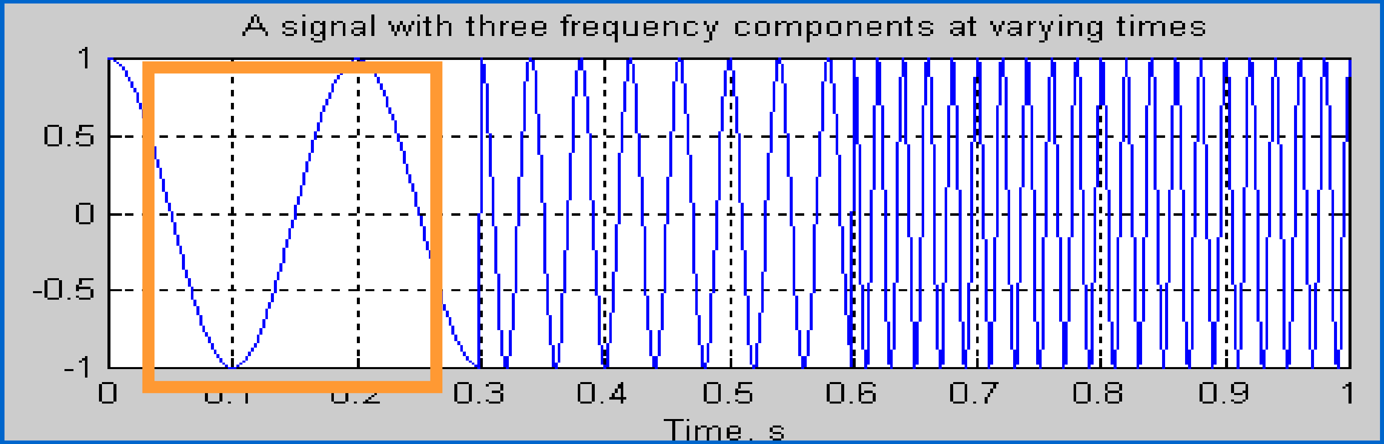
(2) Place the window on top of the signal at t=0

(3) Truncate the signal using this window

(4) Compute the FT of the truncated signal, save results.

(5) Incrementally slide the window to the right

(6) Go to step 3, until window reaches the end of the signal



STFT - Definition in the continuous case

2D function:

Time parameter

Frequency parameter

Signal to be analyzed

$$STFT_f^u(t',u)=\int_t\left[f(t)\cdot W(t-t')\right]\cdot e^{-j2\pi ut}dt$$

STFT of f(t): computed for each window centered at t=t'

Windowing function

Centered at t=t'

Choosing Window W(t)

- What shape should it have?
  - Rectangular, Gaussian, …
- How wide should it be?
  - Window should be narrow enough to ensure that the portion of the signal falling within the window is stationary.
  - But … very narrow windows do not offer good localization in the frequency domain.

STFT Window Size

$$STFT_f^u(t',u)=\int_t\left[f(t)\cdot W(t-t')\right]\cdot e^{-j2\pi ut}dt$$

***W(t)* infinitely long**: $W(t)=1$ → STFT turns into FT, providing excellent frequency localization, but no time localization.

***W(t)* infinitely short**: $W(t)=\delta(t)$ → results in the time signal (with a phase factor), providing excellent time localization but no frequency localization.

$$STFT_f^u(t',u)=\int_t\left[f(t)\cdot \delta(t-t')\right]\cdot e^{-j2\pi ut}dt = f(t')\cdot e^{-jut'}$$

STFT Window Size (cont'd)

- Wide window → good frequency resolution, poor time resolution.
- Narrow window → good time resolution, poor frequency resolution.
- Wavelets (next year): use multiple window sizes.

(1927: Position vs. velocity of an object)

Heisenberg (or Uncertainty) Principle

$$\Delta t \cdot \Delta f \geq \frac{1}{4\pi}$$

Time resolution: How well two spikes in time can be separated from each other in the frequency domain.

Frequency resolution: How well two spectral components can be separated from each other in the time domain

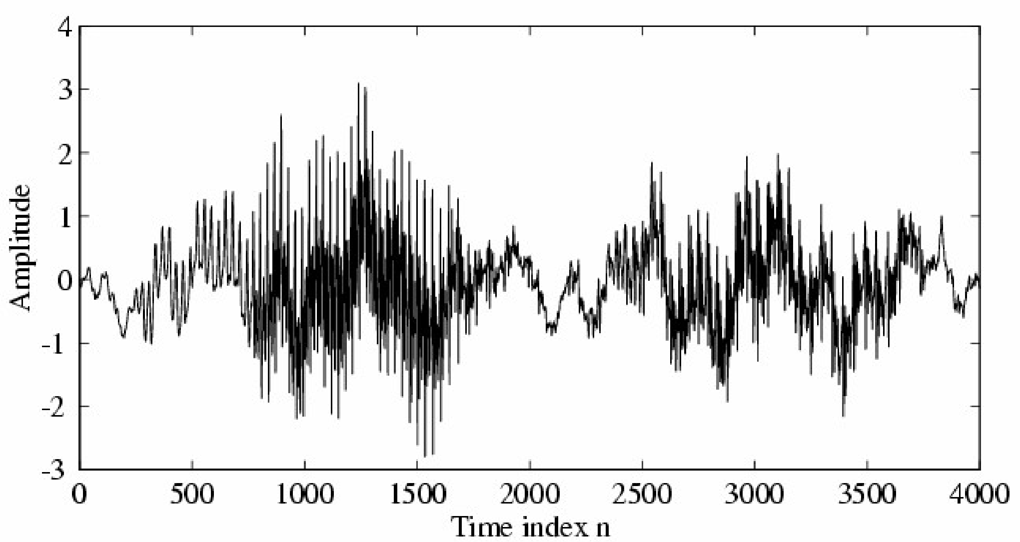
Δt *and* Δf *cannot be made arbitrarily small!*

Heisenberg (or Uncertainty) Principle

- We cannot know the exact time-frequency representation of a signal.
- We can only know what *interval of frequencies* are present in which *time intervals*.

STFT Computation Using MATLAB

- The M-file `specgram` can be used to compute the STFT of a signal
- The application of `specgram` is illustrated next

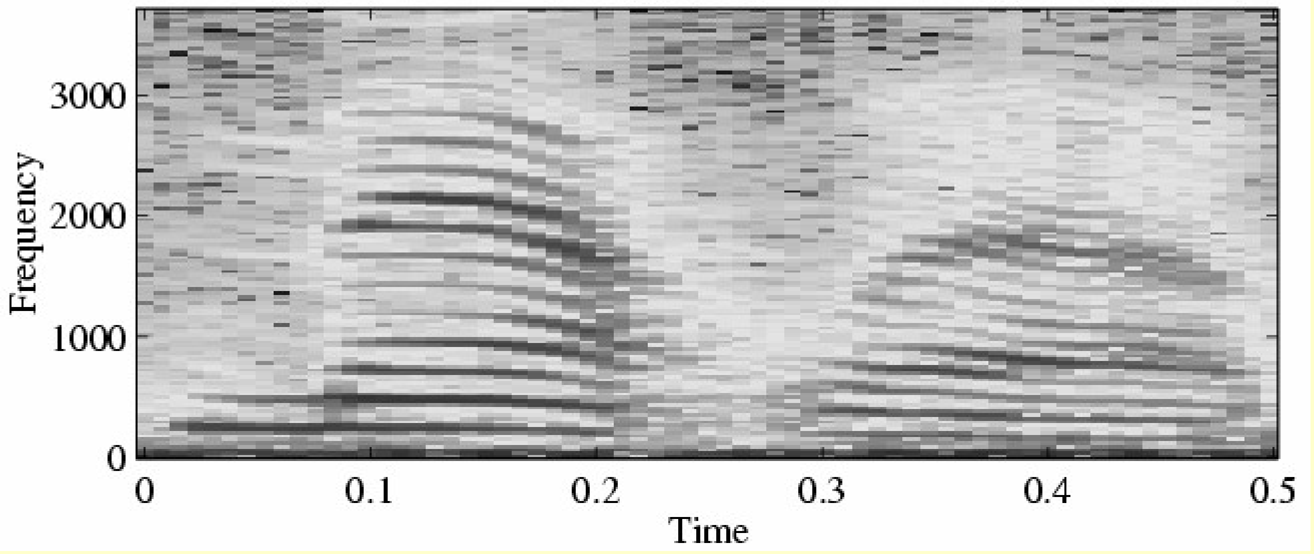


A speech signal of duration 4001 samples

STFT Computation Using MATLAB

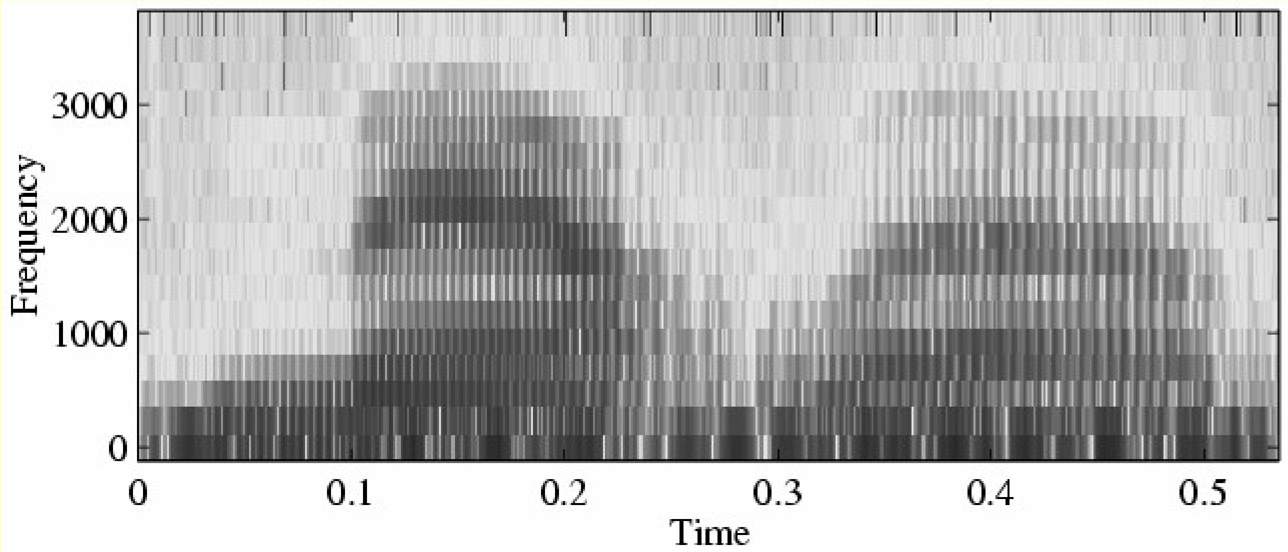
sampling freq. ?

- Using Program 11_4 we compute the **narrowband spectrogram** of this speech signal



STFT Computation Using MATLAB

- The **wideband spectrogram** of the speech signal is shown below



- The frequency and time resolution tradeoff between the two spectrograms can be seen