Morphological image processing

→ Performs suitable *transformations* of images in order to make explicit particular **shape** information.

Mathematical Morphology provides mathematical tools based on *set theory* for investigating and manipulating geometric structures in **binary** images. Image objects are represented as sets.

- The latter may be derived from graylevel images by **segmentation** methods (e.g. using thresholds or edges)
- By the way: some morphological operators, extended to **graylevel** images, can become useful *preprocessing* tools

Morphological image processing

Morphological Operators

Erosion and **dilation** are the most elementary operators of mathematical morphology.

More complicated **morphological operators** can be designed by means of combined erosions and dilations.

Some History

George Matheron (1975) *Random Sets and Integral Geometry*, John Wiley.

Jean Serra (1982) *Image Analysis and Mathematical Morphology*, Academic Press.

Petros Maragos (1985) *A Unified Theory of Translation-Invariant Systems with Applications to Morphological Analysis and Coding of Images*, Doctoral Thesis, Georgia Tech.

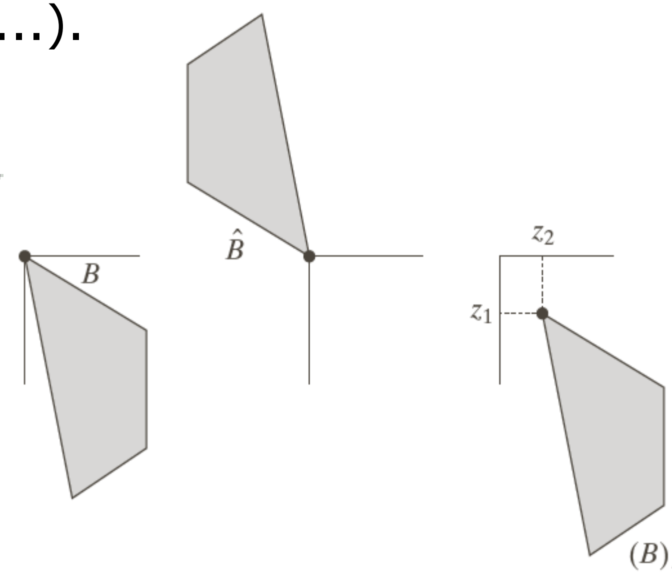
Morphological image processing

Sets of (binary or graylevel) pixels are defined; standard set operations are defined (union, intersection, complement...).
Two more operations are defined:

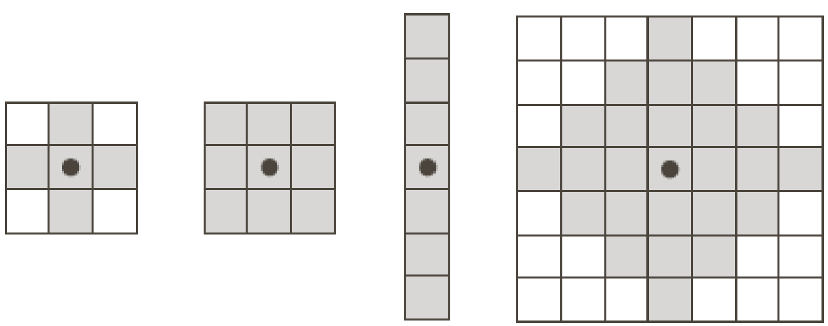
Translation $$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$$

Reflection $$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$$

[*a*,*b*,*w*,*z* are coord. pairs]



Structuring elements (SE) (sort of "probes") operate on the pixel sets

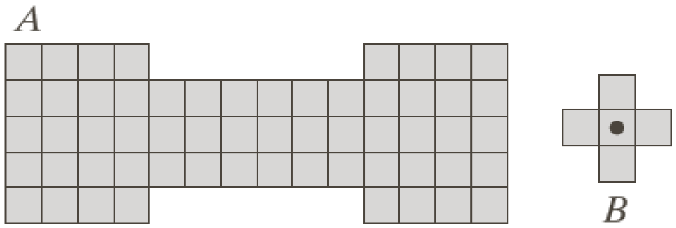


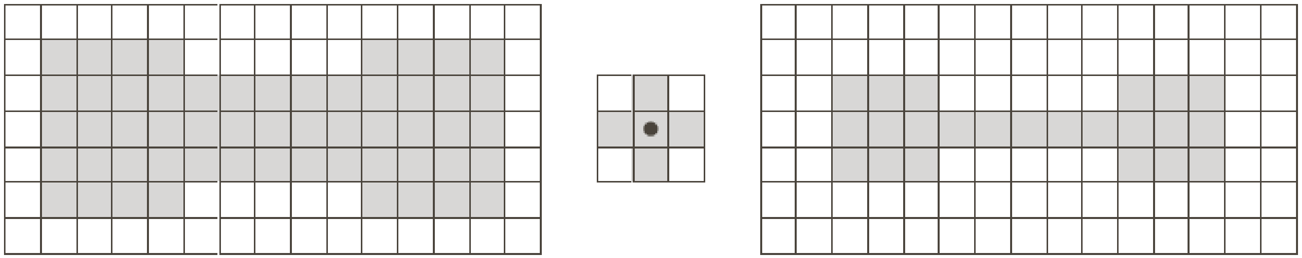
A SE (in gray) can have any shape, can be connected or not. To be used for MM it may need to be padded to form a rectangle. Black dot = origin of SE

Morphological image processing (binary)

E.g.: A binary object *A* is present in an image. The SE *B* runs over the input image. The output set is formed by the positions in which *B* is completely contained in *A* (erosion) ...

[The image needs to be padded to allow for all SE positions]





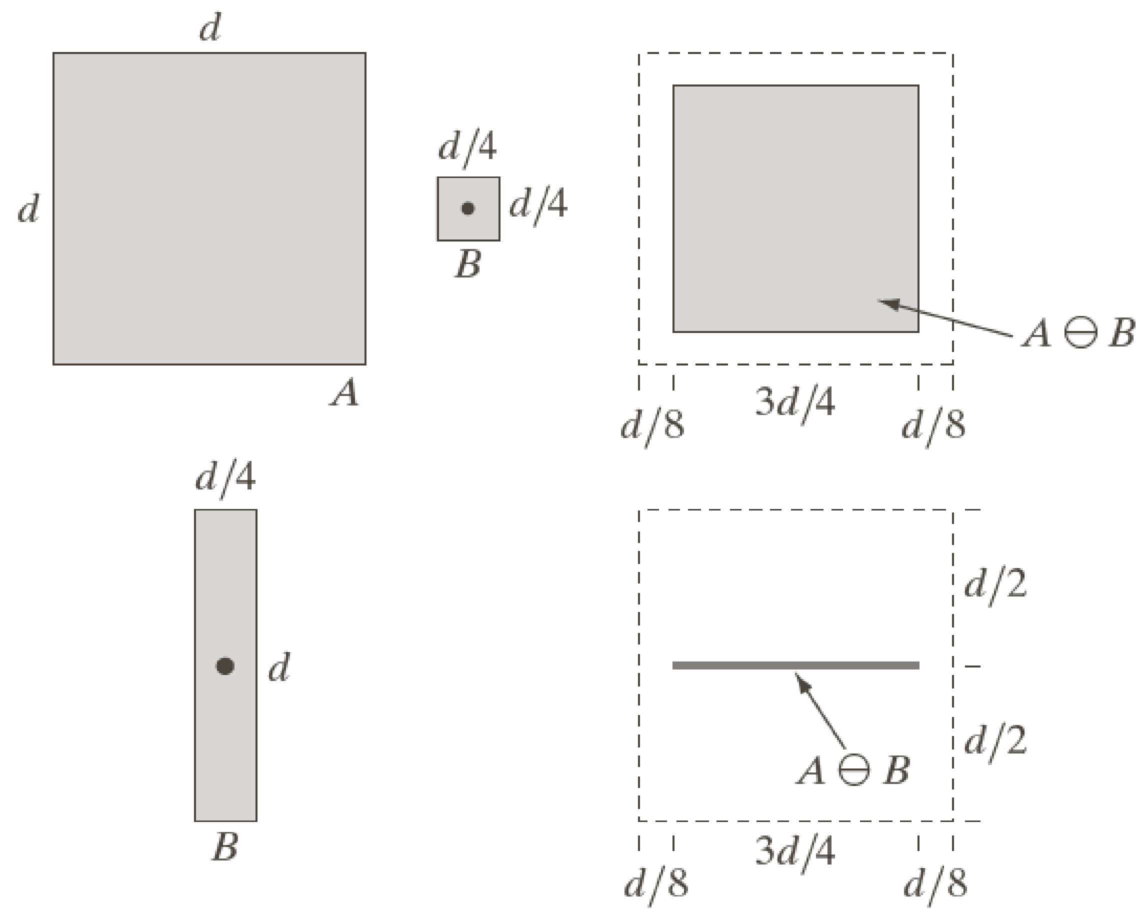
... or, e.g., *B* and *A* overlap by at least one element (dilation)

Morphological image processing (binary)

EROSION

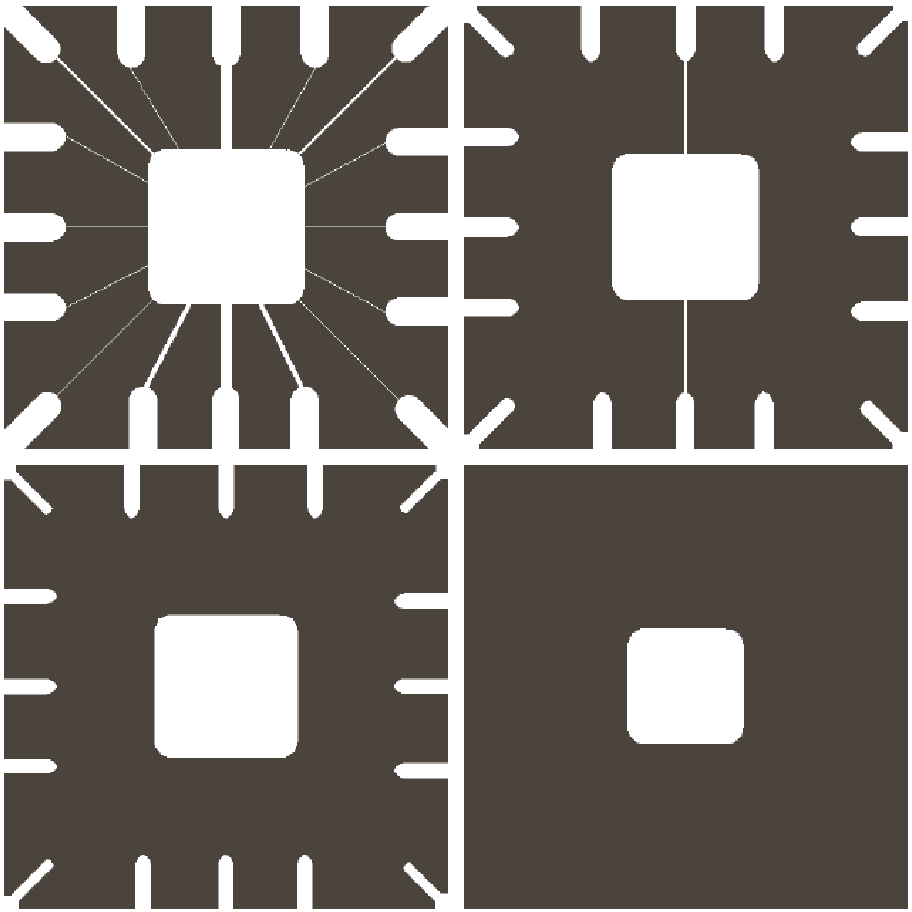
Set of all points *z* such that *B*, translated by *z*, is contained in *A*

$$A \ominus B = \{z \mid (B)_z \subseteq A\} = \{z \mid (B)_z \cap A^C = \emptyset\}$$



Morphological image processing (binary)

EXAMPLE



a	b
c	d

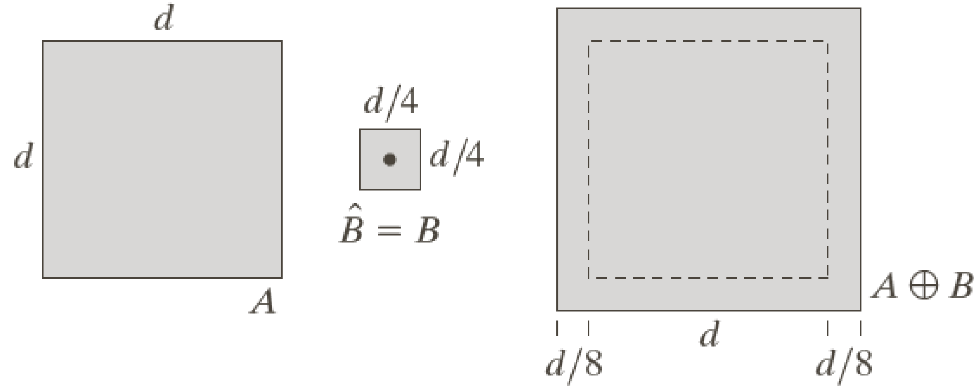
FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Morphological image processing (binary)

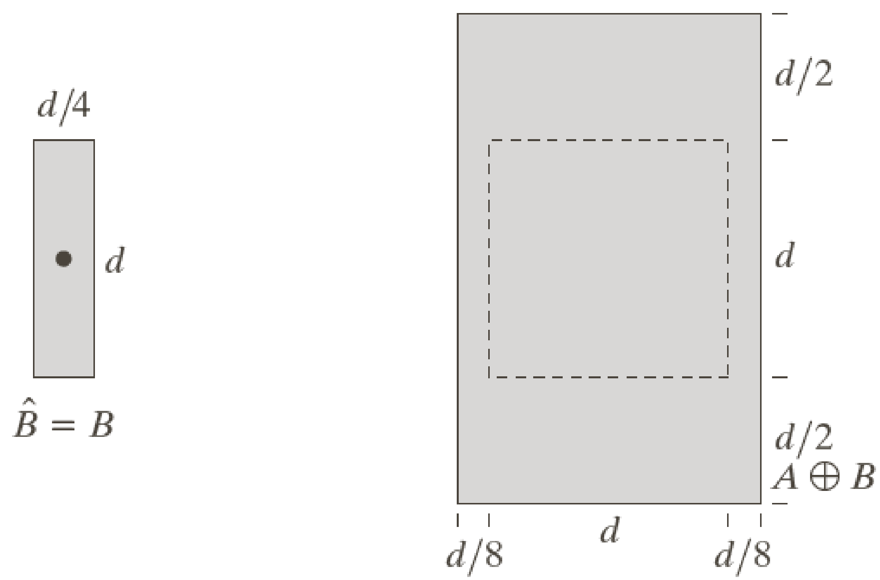
$$A \oplus B = \{z \,|\, (\hat{B})_z \cap A \neq \emptyset\} = \{z \,|\, ((\hat{B})_z \cap A) \subseteq A\}$$

DILATION

Set of all points *z* such that *B*, flipped and translated by *z*, has a non-empty intersection with *A*



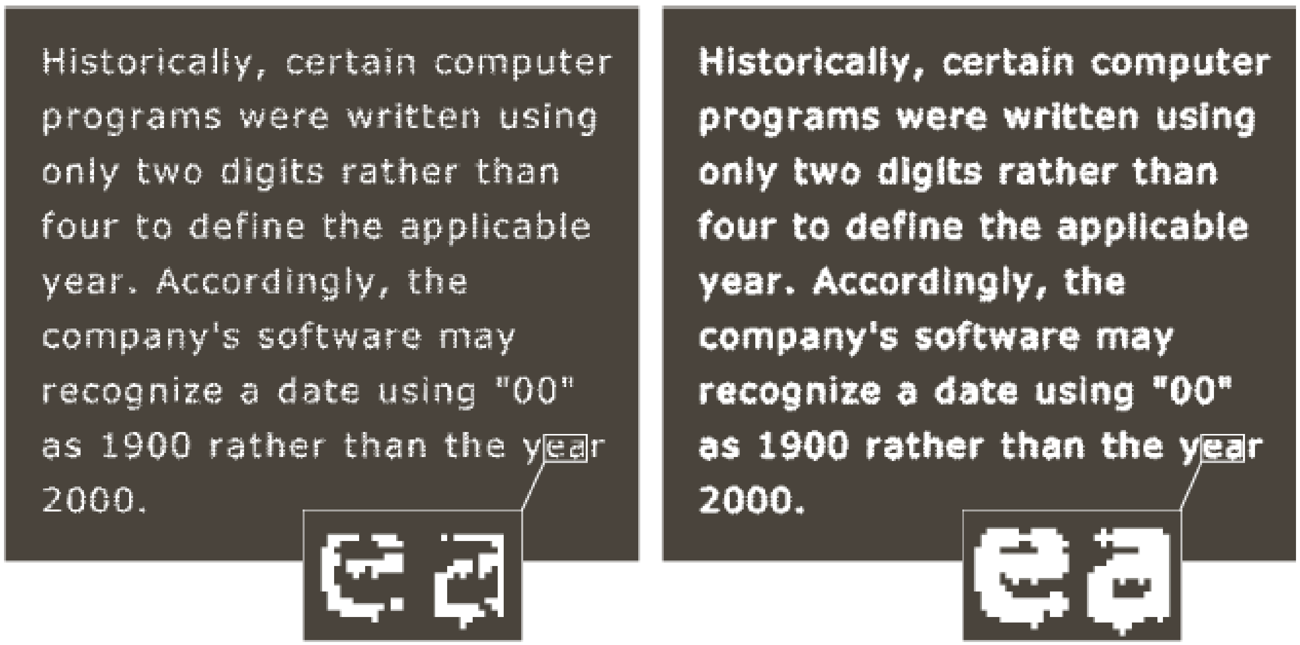
NOTE: the flipping of the structuring element is included in analogy to convolution. Not all Authors perform it



Morphological image processing (binary)

EXAMPLE: fix broken characters using dilation

note: differently from a linear filter, a binary image is output



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Morphological image processing (binary)

EXAMPLE: Eliminate small objects by erosion+dilation

note 1: white objects on black background (opposite wrt prev. slides)

note 2: the final dilation will NOT yield in general the exact shape of the original objects

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Morphological image processing (binary)

OPENING = erosion followed by dilation

$$A \circ B = (A \ominus B) \oplus B$$

- Smoothes convex vertices,
- breaks narrow isthmuses,
- eliminates small islands and sharp peaks

CLOSING = dilation followed by erosion

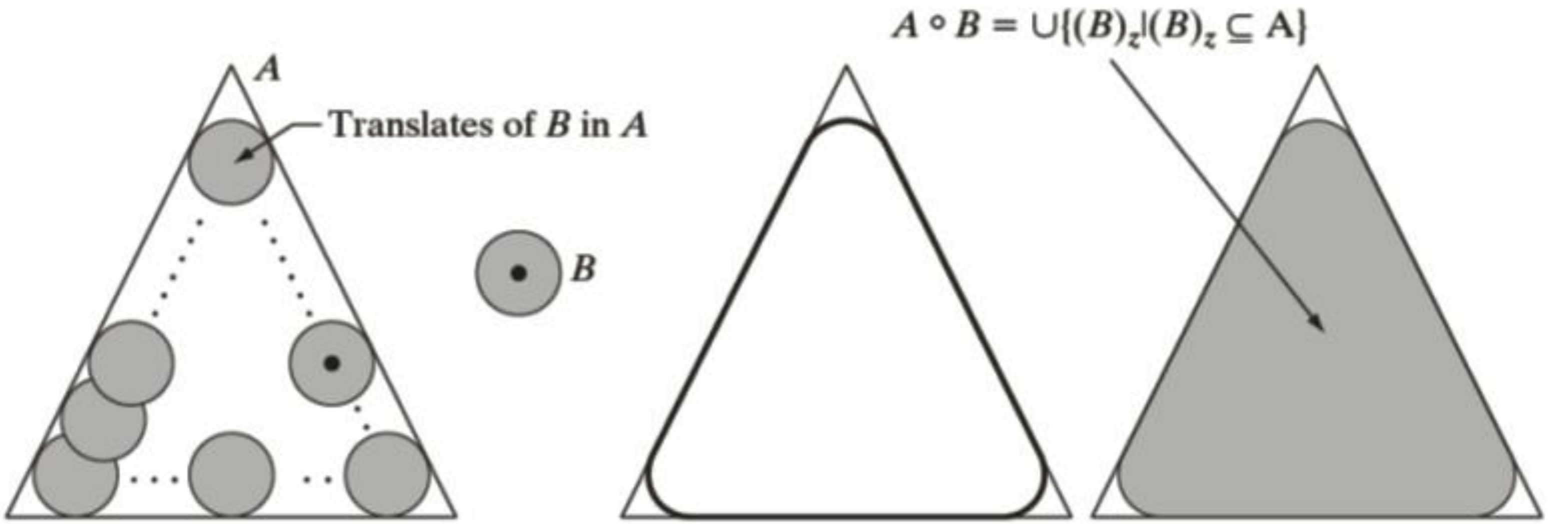
$$A \bullet B = (A \oplus B) \ominus B$$

- Smoothes concave vertices,
- fuses narrow breaks and long thin gulfs,
- eliminates small holes

Morphological image processing (binary)

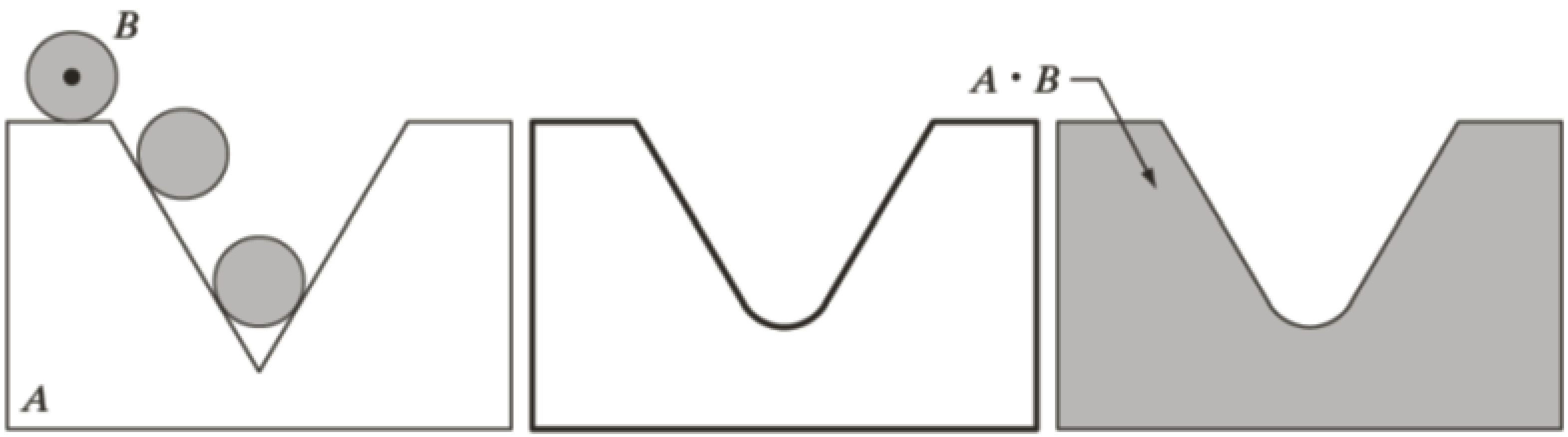
OPENING

shift the SE inside the object border



CLOSING

shift the SE outside the object border



For clarity, object *A* is not shaded in the first picture

Morphological image processing (binary)

To predict the results, compare the size of the SE to the size of the various image sub-structures

OPENING

erode

dilate

CLOSING

dilate

erode

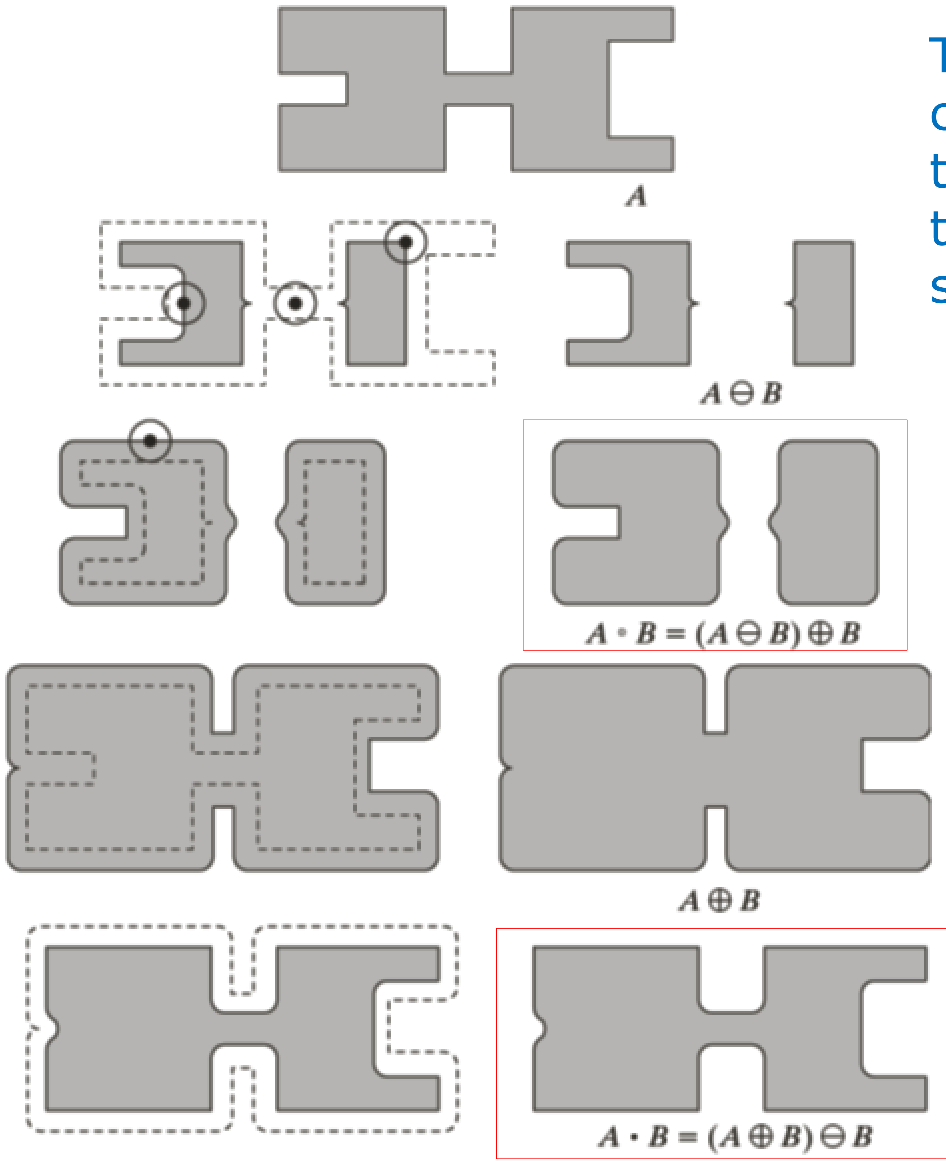


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Morphological image processing (binary)

A property:

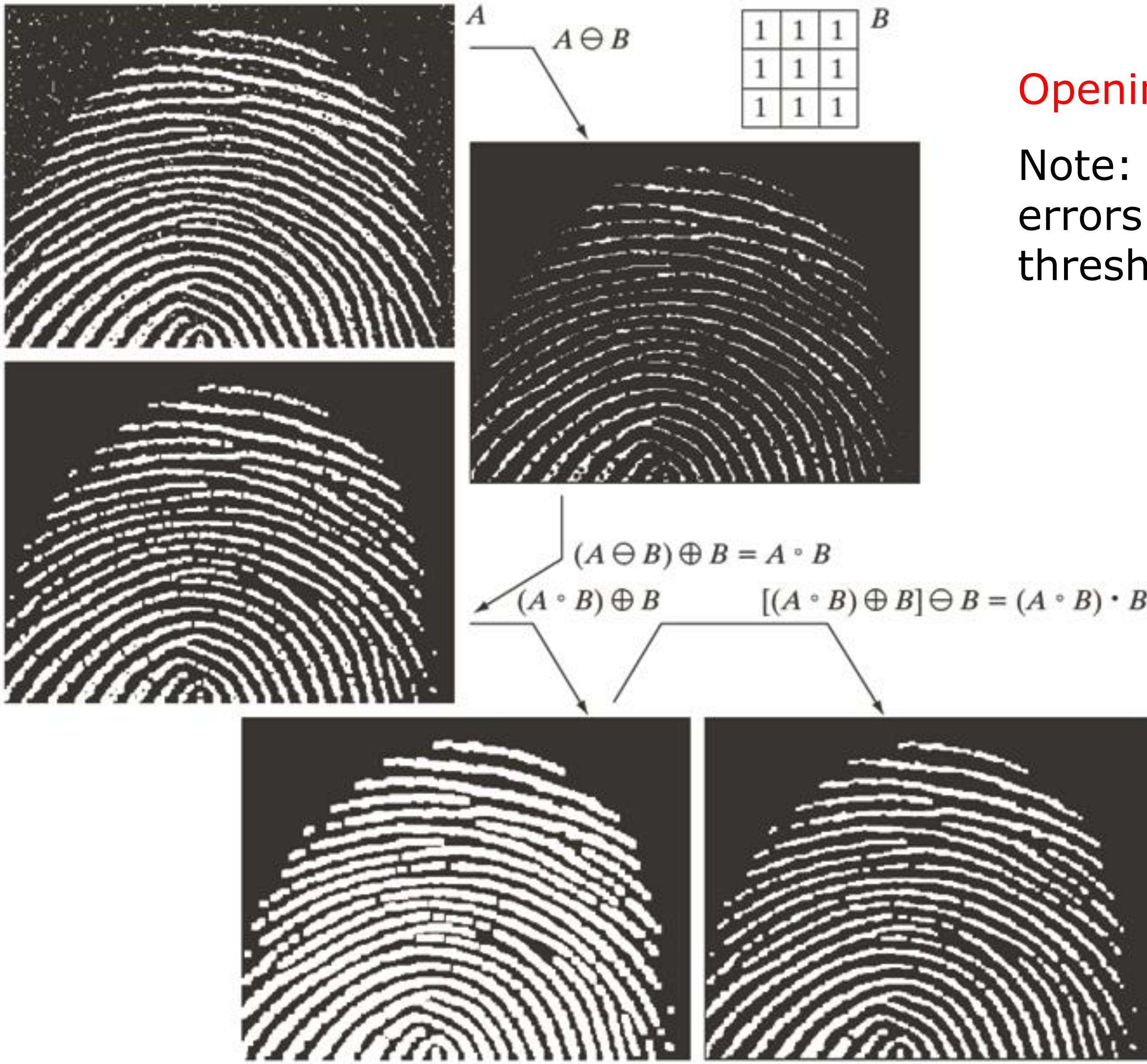
Erosion and Dilation
Opening and Closing

are **dual** operators wrt set complementation and reflection:

$$(A \ominus B)^C = A^C \oplus \hat{B}$$

$$(A \bullet B)^C = A^C \circ \hat{B}$$

Morphological image processing (binary)

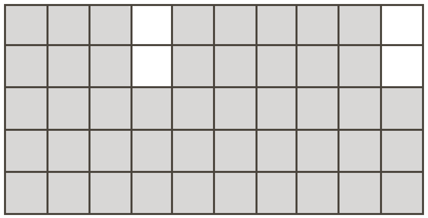


Opening → closing

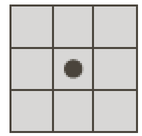
Note: *both* white and black errors (e.g. due to poor thresholding) are eliminated

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Morphological image processing (binary)



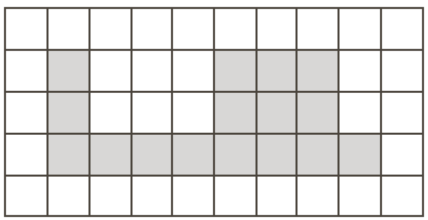
A



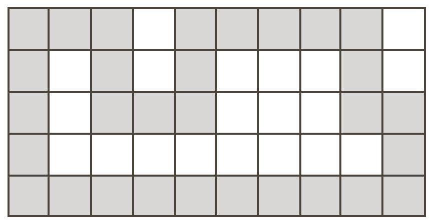
B

BOUNDARY EXTRACTION

$$\beta(A) = A - (A \ominus B)$$

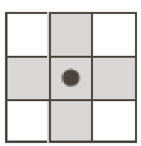


$A \ominus B$



$\beta(A)$

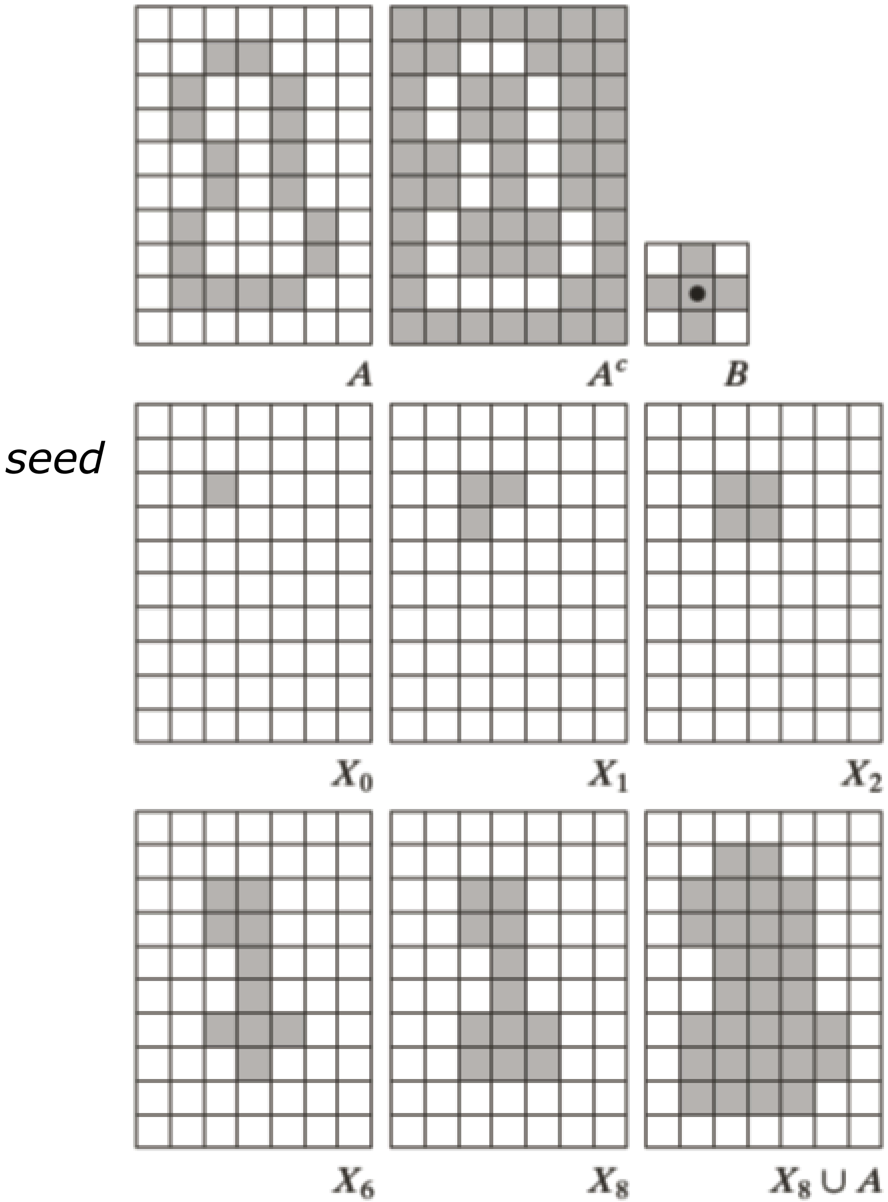
Note: use



to get an 8-connected border

(use a 5x5 SE to get a 2-pixel-wide border)

Morphological image processing (binary)



HOLE FILLING

$$X_0 = P$$

while $X_k \neq X_{k-1}$ do

$$X_k = (X_{k-1} \oplus B) \cap A^C$$

end

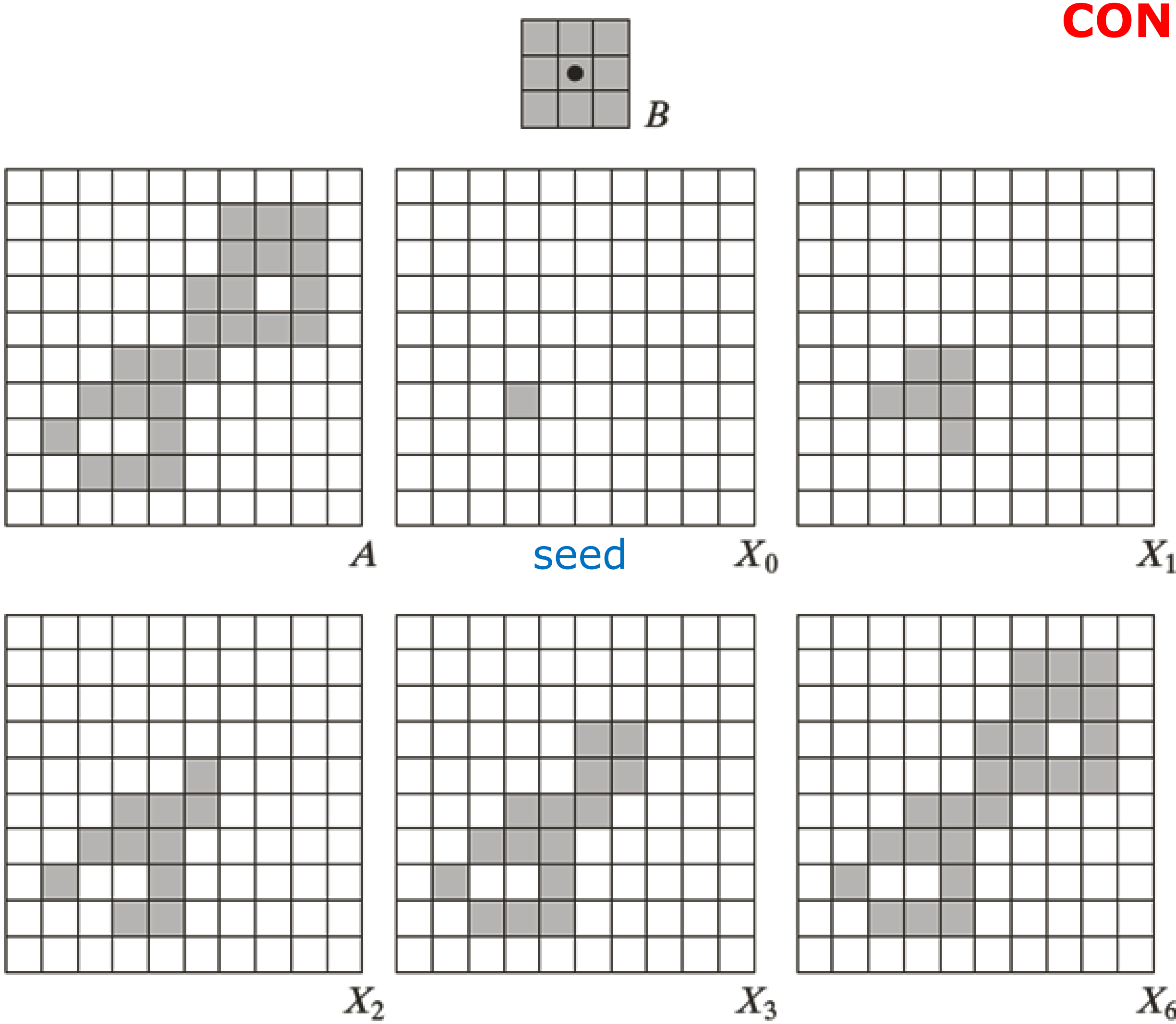
$$X_F = X_k \cup A$$

The dilation would fill the whole area were it not for the intersection with A^C

We call it **Conditional dilation**

Morphological image processing (binary)

EXTRACTION OF CONNECTED COMPONENTS



$$X_0 = P$$

while $X_k \neq X_{k-1}$ do

$$X_k = (X_{k-1} \oplus B) \cap A$$

end

It is again a conditional dilation, with *A*

SE *B* provides 8-connected object

SE in previous slide for 4-connected obj.

Morphological image processing (binary)

GEODESIC DILATION AND EROSION

Note: In its original sense, a *geodesic* is the shortest route between two points on the Earth's surface, namely, a segment of a large circle. It is a generalization of the notion of a straight line to curved spaces

In *dilation*, structures grow at their boundaries. To constrain their growth within some predefined boundaries we use

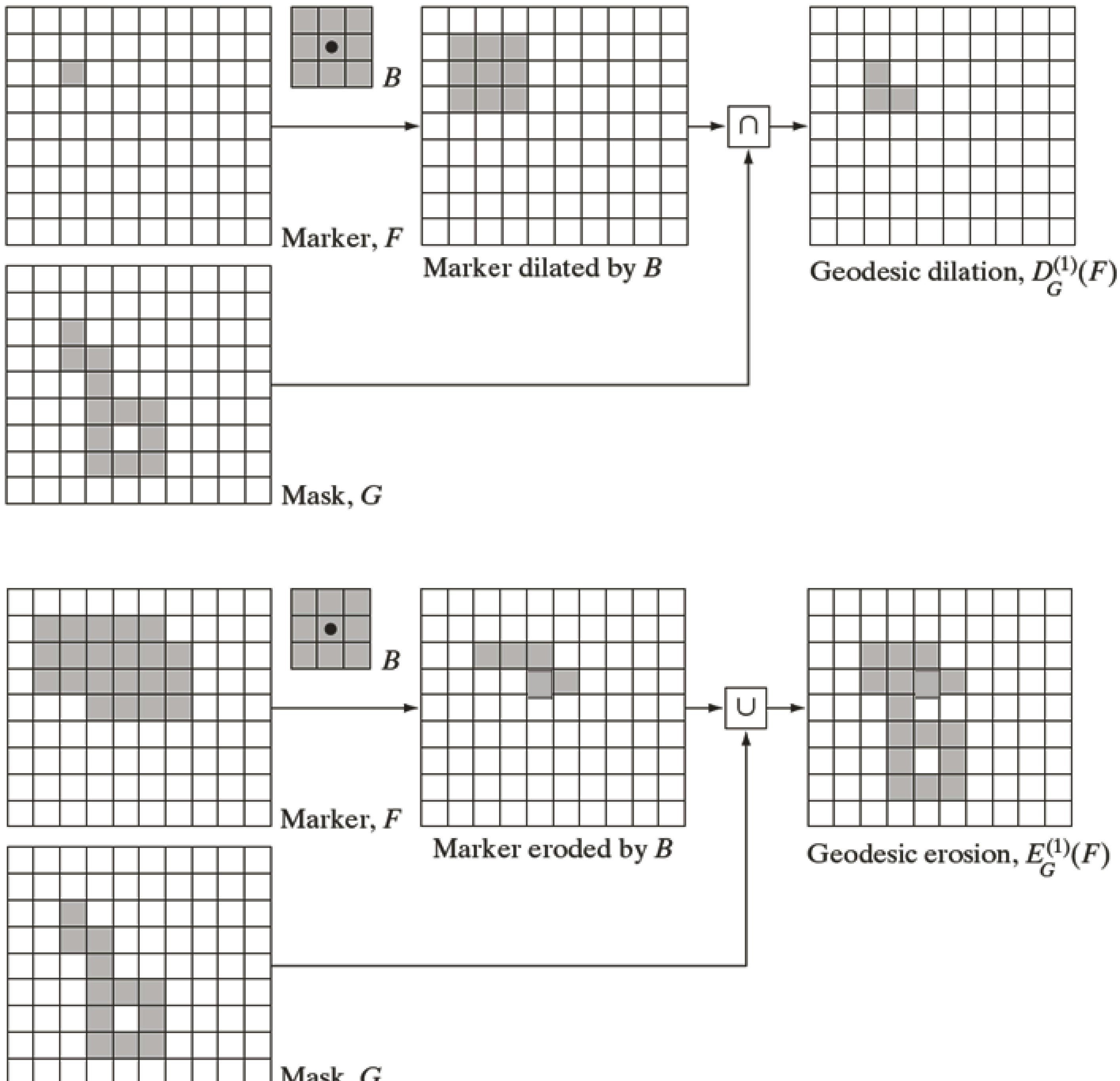
- ***Geodesic dilation***: the minimum between the dilation of the image *(marker, F)* by an SE *(B)* and a control image *(mask, G)* that contains the restraining boundaries
- ***Geodesic erosion*** is similarly defined as the maximum between the erosion of the *marker F* and the *mask G*

$$D^{1}_{G}(F) = (F \oplus B) \cap G$$

$$E^{1}_{G}(F) = (F \ominus B) \cup G$$

Morphol

Geodesic dilation and erosion



Morphological image processing (binary)

Geodesic dilation and erosion can be iterated until convergence (that always occurs) → Reconstruction by dilation, Reconstruction by erosion

As we know, erosion + dilation = opening.

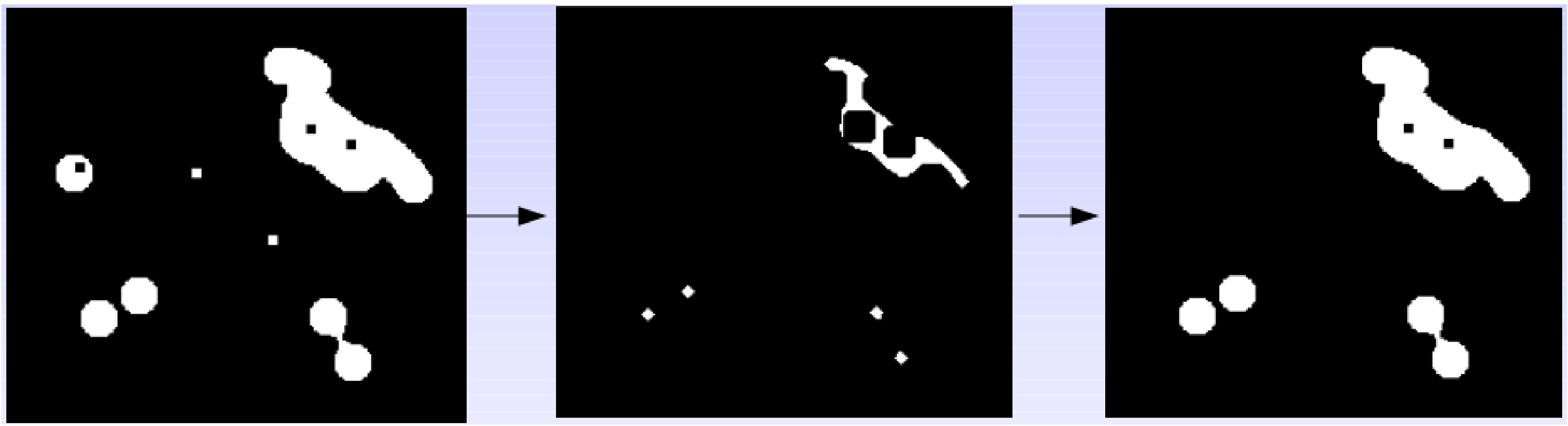
Erosion + reconstruction by dilation is called **Opening by reconstruction**

- Erosion removes small structures, and erodes the boundaries of large structures. Parts of the latter remain present in the eroded image.
- A reconstruction by geodesic dilation, *with the original image as the control image (mask)*, makes these parts grow back to their original size.
- The small structures that were completely removed by the erosion will not grow back → Opening by reconstruction removes small structures, keeping larger structures intact

Dilation + reconstruction by erosion is called **Closing by reconstruction**

Morphological image processing (binary)

Opening by reconstruction



Closing by reconstruction

Morphological image processing (binary)

Opening by reconstruction

Extract characters that contain long vertical strokes

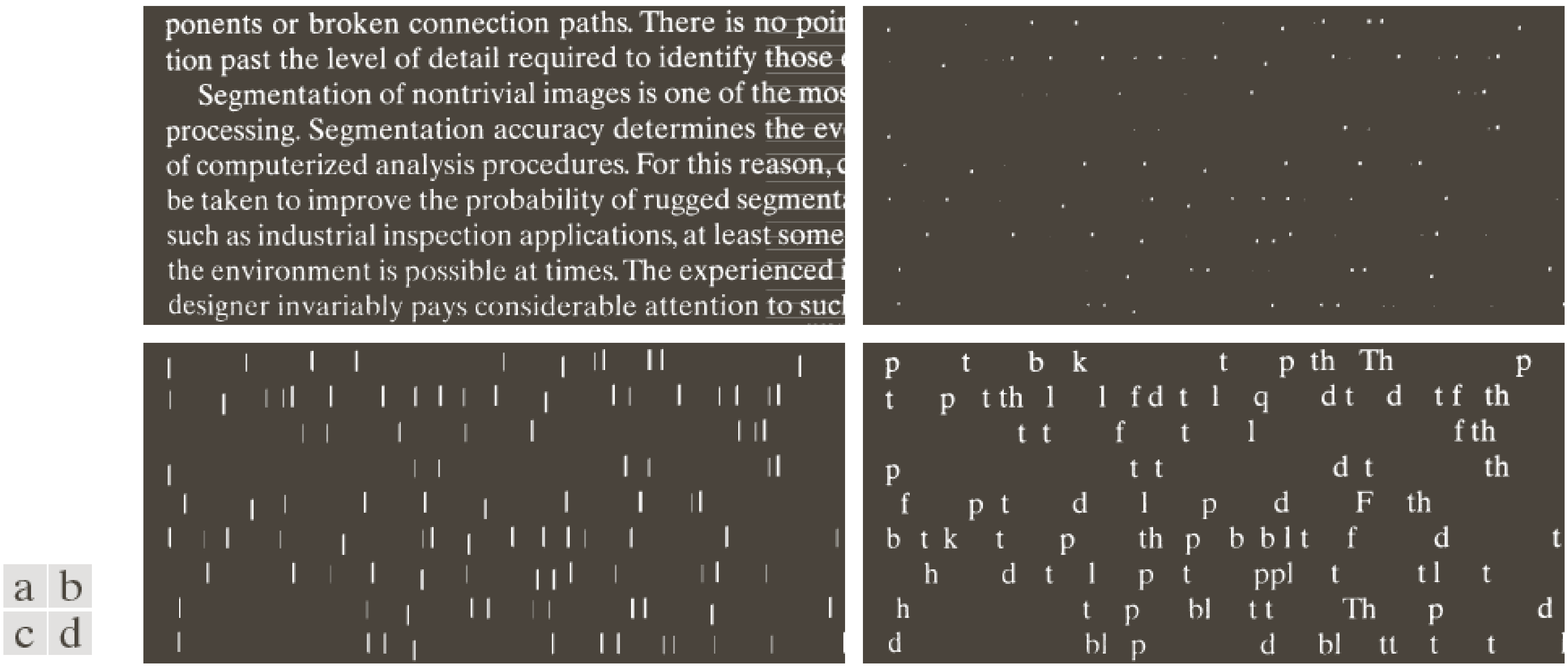
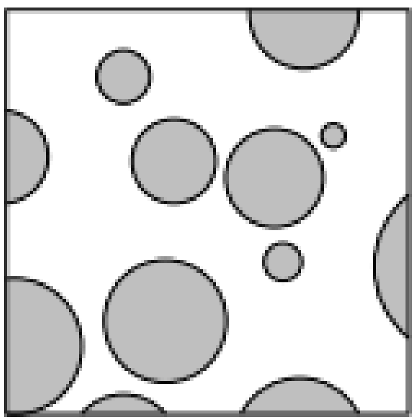


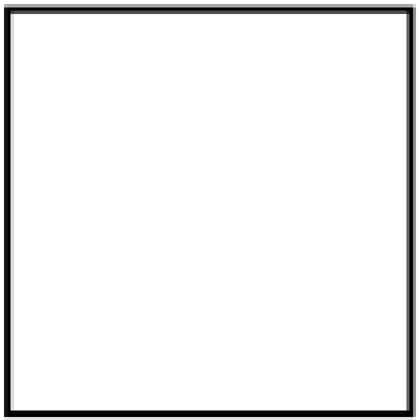
FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

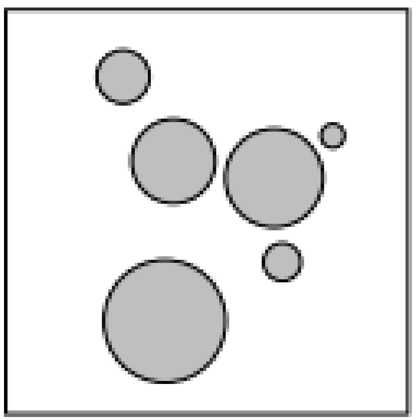
Morphological image processing (binary)

BOUNDARY CLEANING

Delete objects that touch the boundary of the image (probably incomplete objects)





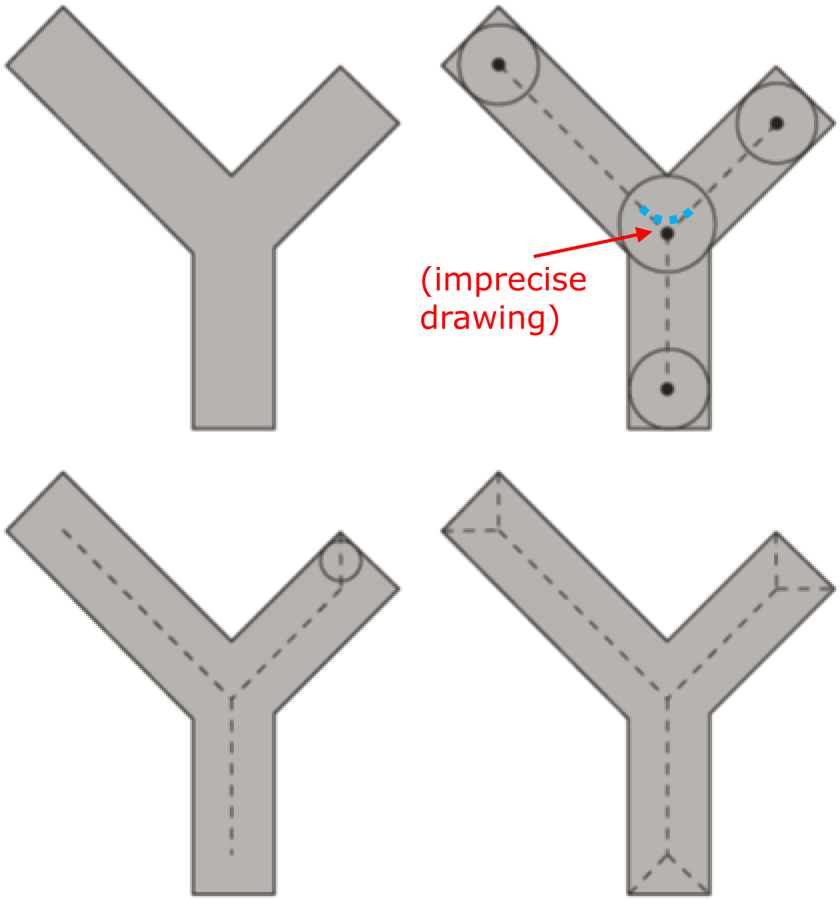


Set marker image as the border of the original image; compute reconstruction by dilation; subtract the result from the image

Morphological image processing (binary)

SKELETONIZATION

- *Maximum disk*: largest disk included in *A*, touching the boundary of *A* at two or more different places
- *Skeleton*: set of the centers of the maximum disks



A prairie fire approach can also be used:

the boundary of the object is set on fire and the skeleton is the loci where the fire fronts meet and quench each other.

Morphological image processing (binary)

SUMMARY

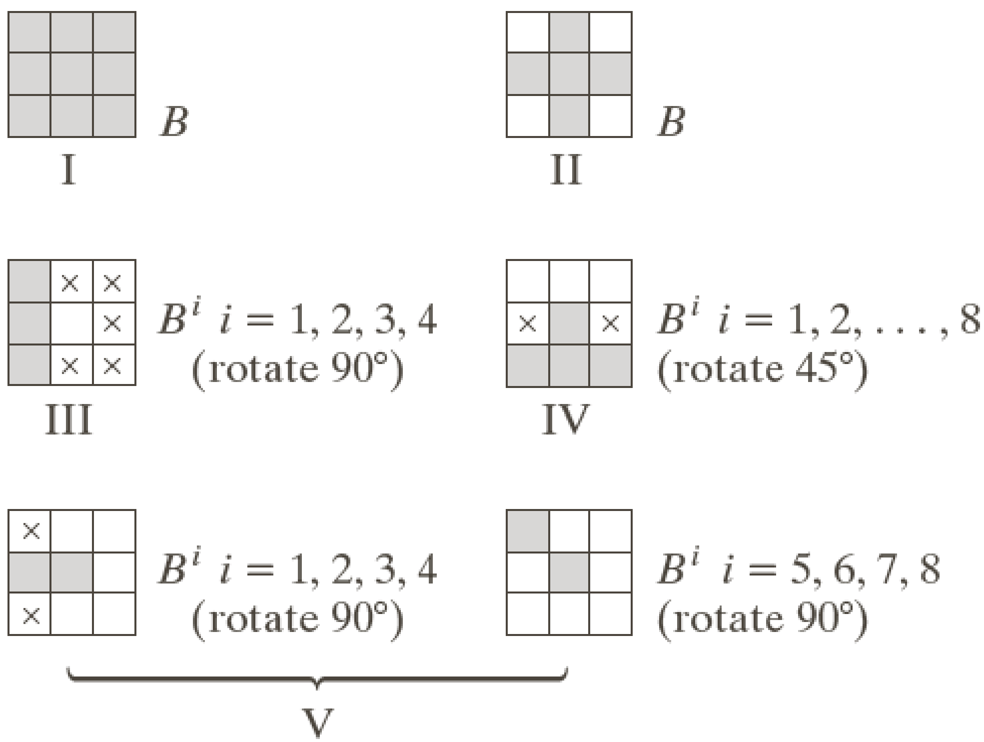


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

Morphological image processing (binary)

SUMMARY

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(Continued)

Morph

SUMMARY

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

Morph

SUMMARY

Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

Morph

SUMMARY

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	

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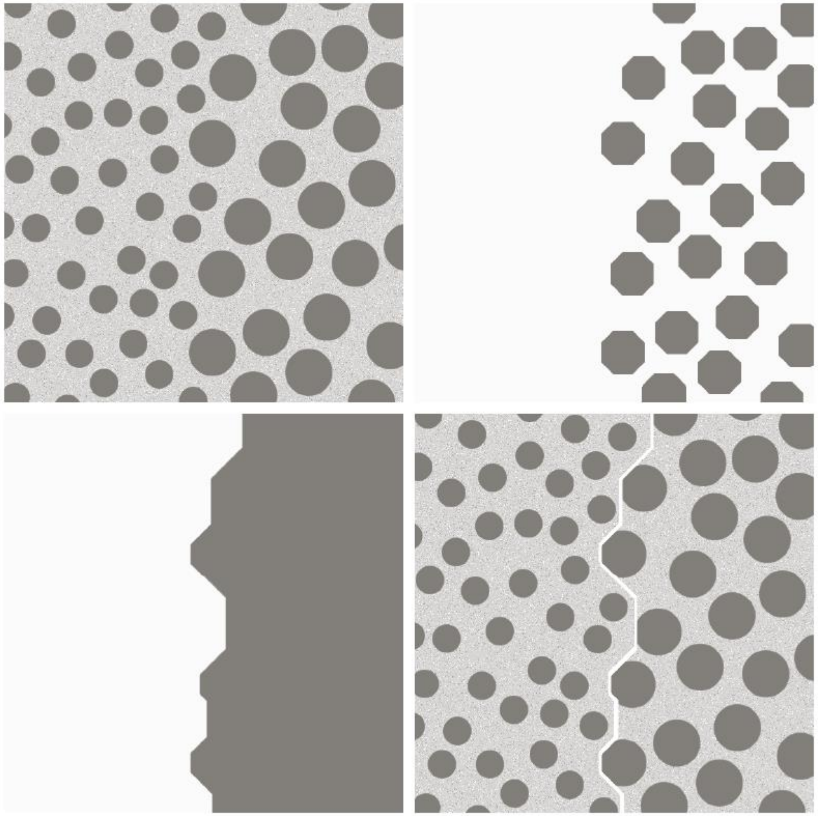
SUMMARY

Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

Morphological image processing (**graylevel** or binary)

Texture segmentation



(a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

See MM concepts, examples, and Matlab commands: https://it.mathworks.com/help/images/morphological-filtering.html