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A common cause model for quantum correlations

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Starting from algebraic results of Hofer-Szabó, Szabó and Rédei on Reichenbach's common cause principle and its applicability, and starting from Suárez and San Pedro's work on the Principle of Common Cause and indeterminism, a causal, local and non-conspiratorial solution for EPR/Bohm correlations is offered. This solution enforces Cartwright's idea that the best way to explain quantum correlations causally and locally is to adopt, for non deterministic contexts, a non-Reichenbachian common cause model, namely the *general fork* model.

1. Introduction

Two decades ago, the debate about causal inference in quantum realm became particularly lively and a large part of the literature focused on an attempt to provide a causal and *local* explanation for EPR/Bohm correlations.
The meaning of the word "local" is commonly expressed by the condition known as *factorizability*. John Bell used this condition to derive his inequali-

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ties[1]. As is well known, quantum mechanics predicts a violation of Bell's inequalities, so that any causal model for quantum correlations must be a non-factorizable model.

There is a widespread belief that any causal model for quantum correlations must be non-local, since it must be non-factorizable. However, it is not true that any non-factorizable model is a non-local model.

2. EPR/Bohm correlations: a brief overview

In Bohm's version of the EPR experiment[2], two entangled particles (1 and 2) of spin ½ are emitted by a source and they move away in opposite directions. The total state of this composite system is known as *singlet state* and it can be formalized as follows:

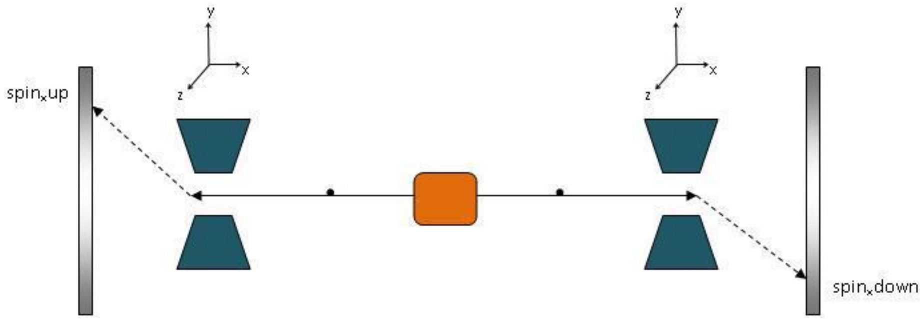
$$\Psi_S^{1,2} = \frac{1}{\sqrt{2}}\left[\uparrow_\theta^1 \downarrow_\theta^2 - \downarrow_\theta^1 \uparrow_\theta^2\right] \tag{1}$$

In each wing a measurement of the spin is performed by means of a Stern-Gerlach magnet in one of three different directions θ = (x, y, z). In this setup, the particles at each wing of the experiment will be observed to have either spin-up (↑) or spin-down (↓) along a given direction (with probability ½ respectively).

Furthermore, it is assumed that the events occurring in one wing of the experiment are space-like separated from those occurring in the other wing, so that causal connections between the different distant events are usually ruled out.

When measurements are taken in both wings with the magnets set on the same direction, i.e. for parallel settings, we will observe what is known as *perfect correlations*, that is perfect correlations of opposite results. In such cases, if the particle 1 has spin-up for an arbitrary direction of the magnet, then the particle 2 has spin-down for the same direction with probability 1, and vice versa. Equally, given parallel settings we never have a situation where both particle 1 and the particle 2 have spin-up (spin-down).

[1] Bell (1964).
[2] Bohm (1951).



3. The Strong Locality condition

The meaning of the word "local" is commonly expressed by the condition known as *factorizability*. Following Jarrett[3], I will call this condition *strong locality*.

Strong locality:

(2) $$p(L_i^a \wedge R_j^b / V_q \wedge L_i \wedge R_j) = p(L_i^a / V_q \wedge L_i)\, p(R_j^b / V_q \wedge R_j)$$ [4]

This condition maintains that the probability of a measurement outcome in one wing of the experimental setting is independent of any measurement outcome and of the state of the Stern-Gerlach magnet in the other wing of the experimental setting.

Strong locality is the conjunction of two other conditions, which Jarrett calls *completeness* and *locality*.

Completeness:

(3) $$p(L_i^a \wedge R_j^b / V_q \wedge L_i \wedge R_j) = p(L_i^a / V_q \wedge L_i \wedge R_j)\, p(R_j^b / V_q \wedge L_i \wedge R_j)$$ [5]

According to this condition the measurement outcome in one of the two subsystems is independent of the measurement outcome in the other subsystem.

[3] Jarrett (1984).

[4] L_i^a: measurement outcome along the direction *i* in the left wing of the experimental setting
R_j^b: measurement outcome along the direction *j* in the right wing
V_q: hidden variable
L_i: measurement operation in the left wing
R_j: measurement operation in the right wing

[5] This condition is better known as *outcome independence*.

Locality:

(4) $$p(L_i^a/V_q \wedge L_i \wedge R_j) = p(L_i^a/V_q \wedge L_i)$$

(5) $$p(R_j^b/V_q \wedge L_i \wedge R_j) = p(R_j^b/V_q \wedge R_j)$$[6]

According to this condition, the outcome in one of the two subsystems is independent of the measurement performed in the other subsystem.

4. EPR correlations and Reichenbach's common cause model

In a well known article, Bas van Fraassen[7] sheds light on the close relationship between Bell's hidden variables and Reichenbachian common causes. In fact *completeness* is nothing more than Reichenbach's screening-off condition[8].

Van Fraassen derives Bell's inequalities and, as quantum mechanics predicts a violation of Bell's inequalities and since the possibility of a direct interaction between the two subsystems in the EPR/Bohm experiments is excluded, he arrives at the conclusion that no causal model can fit the phenomena that violate Bell's inequalities.

However, the contemporary literature considers van Fraassen's conclusion premature.

Starting from the assumption that van Fraassen's Reichenbachian common causes are not adequate, Szabó tried to provide a local model and to avoid Bell's inequalities using a particular kind of Reichenbachian common causes, known as *separate*-common causes[9].

A *separate*-common cause explanation for EPR/Bohm correlations consists in finding *different* common causes for each correlation:

$$C_{ij}^{ab}$$

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6 This condition is better known as *parameter independence*.

7 Van Fraassen (1982).

8 According to Reichenbach's *conjunctive fork* model (Reichenbach, 1956):

$$p(A \wedge B/C) = p(A/C)\, p(B/C)$$

This condition is known as *screening-off*.

9 Szabó (2000).

Most of the causal explanations for quantum correlations consist in finding *common*-common causes[10].

The notion of *common*-common cause, as it is defined by Hofer-Szabó, Rédei and Szabó, means that a particular common cause may screen off more than one correlation[11].

Szabó's causal model avoids Bell's inequalities, but it turns out to be conspiratorial at a deeper level, that is the measurement operations seem to be statistically correlated with different algebraic combinations of the postulated Reichenbachian *separate*-common causes[12].

5. Reichenbach's common cause model and indeterminism

Szabó's results turn out to be very powerful: given Reichenbachian *separate*-common causes, it seems impossible to provide a local and non-conspiratorial causal explanation for EPR/Bohm correlations for deterministic or stochastic cases.

As put in light by San Pedro and Suárez[13], in non-deterministic cases[14] the expression demonstrated by Suppes and Zanotti[15], according to which screening-off (SO) common causes of perfect correlations (PCORR) are deterministic common causes (DCC), can be rewritten as follows:

(6) $$\neg \mathrm{DCC} \rightarrow \neg(\mathrm{PCORR} \wedge \mathrm{SO})$$

That is:

(7) $$\neg \mathrm{DCC} \rightarrow (\mathrm{PCORR} \wedge \neg \mathrm{SO}) \vee (\neg \mathrm{PCORR} \wedge \neg \mathrm{SO}) \vee (\neg \mathrm{PCORR} \wedge \mathrm{SO})$$ [16]

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[10] Van Fraassen's common causes seem to be *common*-common causes.

[11] Hofer-Szabó, Rédei, Szabó (2000, 2002).

[12] The notion of *conspiratorial* aims to capture the intuition that there is no statistical independence between the common causes and the measurement settings:

$$p(C/L_i \wedge R_j) \neq p(C)$$

[13] San Pedro, Suárez (2008).

[14] A common cause is called *deterministic* if it determines its effects with probability 1, *indeterministic* if not.

[15] Suppes, Zanotti (1976).

[16] *Non-perfect correlations* ($\neg$PCORR) does not mean that the maximal correlations are violated by arbitrary small deviations, but it describes what happens when the experimenters do not choose for both particles the same arbitrary measurement direction θ.

Given (7), it seems more natural to provide a non-screening off *separate*-common causes explanation for indeterministic contexts.

Moreover, the postulated common causes for quantum correlations can be also *common*-common causes. We can imagine a single *common*-common cause for all the correlations. This common cause could be an event *C* at the source, maybe the initial state, that is the singlet state, or the interaction at the source between the two particles.

Hence, *C* must be considered as a single *common*-common cause for all the correlations and it must act not only on the estimated perfect correlations, but also on the non-perfect correlations.

The existence of non-perfect correlations limits us to suppose a context where the common cause acts non-deterministically:

(8) $(\neg \text{PCORR} \wedge \text{SO}) \rightarrow \neg \text{DCC}$

(9) $(\neg \text{PCORR} \wedge \neg \text{SO}) \rightarrow \neg \text{DCC}$

The common cause needs to be indeterministic and, since we have only a single common cause for all the correlations, it also needs to be indeterministic in the case of perfect correlations.

Moreover, as we have already seen, the following principle holds in cases of non-deterministic common causes:

(10) $\neg \text{DCC} \rightarrow (\text{PCORR} \wedge \neg \text{SO})$

Since we have only a single common cause for all the correlations, *C* also needs to be an indeterministic non-screener off in the case of non-perfect correlations.

Furthermore, according to Hofer-Szabó, Rédei and Szabó, *it can always be found an extension for a Reichenbachian common cause incomplete probability space so that it contains Reichenbach's separate-common causes for all the original correlations*[17]. However, this is not true for *common*-common causes[18].

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[17] It means that screening-off *separate*-common causes can be always found even for perfect correlations.

[18] Hofer-Szabó, Rédei, Szabó (2000).

It follows that, for indeterministic contexts, we should give up the screening- off condition, both for the case of *separate*-common causes and for the case of a single *common*-common cause.

6. A non-screening off model for EPR/Bohm correlations

What if we try to give up the screening-off condition, proposing a non-factorizable common cause model? We do not derive Bell's inequalities. According to a widespread belief, any causal model for quantum correlations must be non-local, since it must be non-factorizable. However, it is not true that any non-factorizable model is a non-local model.

Here a general definition of "local":

> Locality [...] means that nothing outside the backward light cone is relevant to the production of the effect, in particular neither the setting of the distant apparatus nor the distant outcome.[19]

We can use a non-Reichenbachian common cause model which does not violate our definition of "locality", that is we can suppose the existence of non-Reichenbachian common causes that act in the past light cone of the outcomes.

Given non-deterministic common causes, the relevant non-screening off models that could explain causally and locally EPR/Bohm correlations are:

- Wesley Salmon's *interactive fork*[20]
- Nancy Cartwright's *general fork*[21]

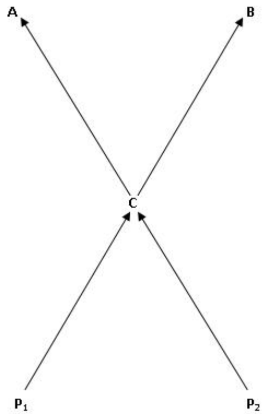
6.1 Salmon's criterion

The *interactive forks* are considered spatio-temporal intersections that violate the screening-off condition, so that:

[19] Cartwright (2008), p. 263.
[20] Salmon (1984).
[21] Cartwright (1987).

(11) $$p(A \wedge B/C) > p(A/C)\, p(B/C)$$

The space-time diagram of the interactive forks has the shape of an *x*:



In this figure P_1 and P_2 are two processes which interact in *C*. *A* and *B* are the two emerging processes. The interaction has the capacity to produce changes in the properties of the two outgoing processes.

Is this the case of EPR/Bohm experiment? In that experiment, we have a molecule consisting of two atoms which are separated after an interaction. But, do the changes in the properties of the two separate particles (atoms) emerge from an interaction between those particles?

It is not at all clear if we have two processes going in and two processes going out, and if the phenomenon we are trying to describe really has an *x*-shape. Maybe we have a single process that bifurcates into two processes and the phenomenon that we are trying to describe has a *y*-shape.

Moreover, I want to draw attention to a condition known as *contiguity condition* and defined as follows:

> Every cause and its effect must be connected by a causal process that is continuous in space and time.[22]

[22] Cartwright, Chang (1993), p. 173.

According to Salmon, the contiguity condition is a necessary condition for any genuine causal model. However, this condition is hard to maintain in some cases treated by quantum mechanics[23].

For the reasons illustrated above, Salmon's criterion does not seem a good model to provide a causal explanation for EPR/Bohm correlations.

6.2 Cartwright's criterion

According to Cartwright's *general fork* criterion, deterministic common causes in general satisfy screening-off, but Reichenbach's screening-off is not valid for genuinely probabilistic cases.

Moreover, Cartwright puts the blame, not only on the indeterministic character of the common causes, but also on the fact that the momentum must be conserved[24].

In order to apply the general fork model to the EPR/Bohm correlations, the common causes must act under some conservation constraint, so that they do not produce their effects independently. In the situation illustrated in the EPR/Bohm experiment, there are two events which occur in tandem, and the angular momentum is to be conserved, that is the singlet state is to be conserved[25].

The best non-screening off model to provide an indeterministic causal explanation for quantum correlations seems to be the general fork model, since the supposed common causes, both in case of single *common*-common causes and in case of *separate*-common causes, act under the constraint of the conservation of the angular momentum, which guides them to make their effects dependent on each other.

Moreover, according to Cartwright, the contiguity condition is not a necessary condition for any genuine causal model[26]. Since the contiguity condition is hard to maintain in quantum mechanics, Cartwright's model seems once again the best non-screening off model to provide a local and causal explanation for EPR/Bohm correlations.

7. Conclusion

According to most of the recent literature, any causal explanation for quantum correlations must be non-local and then incompatible with Special Relativity.

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[23] Cartwright, Chang (1993), p. 178.
[24] Cartwright (1987), p. 184.
[25] Cartwright (1989), p. 232.
[26] Cartwright (2008), p. 262.

However, if being “local” entails that nothing outside its backward light cone is relevant to the production of an effect, then a causal and local explanation can still be found for quantum correlations.

In this paper, I proposed an indeterministic non-Reichenbachian solution to explain causally quantum correlations. This solution can avoid the derivation of Bell's inequalities and, at the same time, provide a local explanation.

REFERENCES

BELL, J.S. (1964): “On the Eistein-Podolsky-Rosen Paradox”, *Physics,* 1, pp. 195-200.

BOHM, D. (1951): *Quantum Theory,* New York: Prentice Hall.

CARTWRIGHT, N. (1987): “How to Tell a Common Cause: Generalizations of the Conjunctive Fork Criterion”, in J.H. Fetzer (Ed.): *Probability and Causality*, Dordrecht: Reidel, pp. 181-188.

CARTWRIGHT, N. (1989): *Nature’s Capacities and their Measurement*, New York: Oxford University Press.

CARTWRIGHT, N. (2008): “Reply to Iain Martel”, in S. Hartmann, C. Hoefer, L. Bovens (Eds.): *Nancy Cartwright’s Philosophy of Science*, New York: Routledge, pp. 246-264.

CARTWRIGHT, N., CHANG, H. (1993): “Causality and Realism in EPR experiment”, *Erkenntnis,* 38, pp. 169-190.

HOFER-SZABÓ, G., RÉDEI, M., SZABÓ, L.E. (2000): “Common Cause Completability of Classical and Quantum Probability Spaces”, *International Journal of Theoretical Physics,* 39, pp. 913-919.

HOFER-SZABÓ, G., RÉDEI, M., SZABÓ, L.E. (2002): “Common Causes are not Common-common Causes”, *Philosophy of Science*, 69, pp. 623-636.

JARRETT, J. (1984): “On the Physical Significance on the Locality Conditions in the Bell Arguments”, *Noûs*, 18, pp. 569-589.

REICHENBACH, H. (1956): *The Direction of Time,* Berkeley: University of California Press.

SALMON, W.C. (1984): *Scientific Explanation and the Causal Structure of the World,* Princeton: Princeton University Press.

SAN PEDRO, I., SUÁREZ, M. (2008): “The Principle of Common Cause and Indeterminism: A Review”, *LSE Contingency and Dissent in Science Technical Report 07/08,* October 2008.

SUPPES, P., ZANOTTI, M. (1976): “On the Determinism of Hidden Variable Theories with Strict Correlation and Conditional Statistical Independence of Observables”, in P. Suppes (Ed.): *Logic and Probability in Quantum Mechanics* , Dordrecht: Reidel, pp. 445-455.

SZABÓ, L.E. (2000): “On Attempt to Resolve the EPR-Bell Paradox via Reichenbachian Concept of Common Cause”, *International Journal of Theoretical Physics*, 39, pp. 911-926.

VAN FRAASSEN, B.C. (1982): “The Charybdis of Realism: Epistemological Implication of Bell’s Inequality”, *Synthese*, 52, pp. 25-38.