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Tarski, Etchemendy And The Manifold Concept Of Logical Consequence

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ABSTRACT. Etchemendy claims that Tarski's definition of logical consequence is conceptually inadequate, since it does not capture any pretheoretic notion. I question Etchemendy's assumption that the concept of logical consequence is uniquely determined by pretheoretic intuitions. Metaphysical and epistemological assumptions and considerations on the nature and the aims of logic essentially contribute to determine whether the concept of logical consequence deserves attention and how it has to be defined.

1. Introduction

Etchemendy ([1990], [2008]) attacks Tarski's definition of logical consequence (LC), claiming that it is both conceptually and extensionally inadequate. I will concentrate on the first criticism: Tarski's definition is conceptually inadequate since it does not capture our pre-theoretic notion of LC.
It is evident that Etchemendy tacitly assumes that there is a determined pretheoretic concept of LC, whose properties are grasped by our intuitions. Oddly enough, this point has not received much attention by his commentators.

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Against Etchemendy, I claim that we have a practice of using logically valid arguments, but there is not a unique legitimate way to describe their characteristics and, so, to describe LC. I do not only claim that we find several different characterizations of LC in different authors, which is obvious. My claim is sharper: the concept of LC is deeply related to more general philosophical points of view (involving metaphysical and epistemological concepts and assumptions on the nature and the aims of logic) and this fact prevents us from maintaining that there is only one legitimate definition of LC. This fact prevents us also from maintaining that every, possibly different, characterization of LC should rely on the set of properties isolated by Etchemendy, on the ground of his own intuitions. In order to understand why LC received different legitimate characterizations we have to consider the different ways in which logic was understood, its different tasks and the different philosophical ideas held by logicians.Capoverso senza rientro. Quest'idea di una medicina che applica pezzi di conoscenze di (e metodi tratti da) altre scienze discende dalla formazione medica stessa. Secondo Uexküll e Wesiack (1991), infatti, i futuri medici apprendono le nozioni sulle strutture e i processi del corpo umano ben prima di entrare in contatto con i pazienti, ossia prima di sperimentare i problemi dell'incontro clinico e della gestione della cura, che sono invece lo scopo ultimo della medicina in quanto tale (Pellegrino e Thomasma 1981). Ciò impedisce loro di interpretare in senso pragmatico il dominio teoretico della biomedicina, base concettuale del loro lavoro. Non solo: spesso li porta a non comprendere quanto il ragionamento e l'intervento in clinica siano "carichi, anzi saturi di teorie" – cioè condizionati da metodologie, epistemologie e persino metafisiche di scienze non mediche, che tuttavia alimentano e indirizzano l'evoluzione della medicina scientifica.

2. Tarski

Tarski ([1936]) begins to sketch his attempt to define LC for a large class of formalized languages with two intuitive considerations. Let K be a class of sentences and X a sentence of the same formal language which follows logically from K. The two intuitive considerations are

(a) truth preservation: it can never happen that both the class K consists only of true sentences and the sentence X is false

(b) formality: the relation of LC cannot be influenced by knowledge of the objects to which the sentence X or the sentences of the class K refer.

Assumed a formal language L, to give a precise definition encompassing these properties, Tarski introduces the notions of satisfaction of a sentential function by a sequence of objects and of model. A model of a class K of sentences is a sequence of objects that satisfies all the sentential functions obtained by uniformly replacing all the non-logical symbols in the sentences in K by corresponding variables. A model of a sentence is a model of the class whose only member is that sentence. Thus, for every class K of sentences and for every sentence X, Tarski's definition of LC is:

> The sentence X follows logically from the sentences of the class K iff every model of the class K is also a model of the sentence X (p. 417).

This definition ensures that we take into consideration every possible permutations of objects. So Tarski claims that his account satisfies the requirements above.

3. Etchmendy

According to Etchemendy, Tarski proposes a *reductive* account of LC but he does not capture the properties surrounding the proper concept of LC nor does he provide any assurance that his account will declare valid an argument iff this argument is indeed valid ([2008] p. 265-7). Tarski's account reduces LC to material consequence and generalization since it establishes that an argument is logically valid iff, for every sequence of objects, if the premises are satisfied, then the conclusion is satisfied (*ivi* p. 265). Why does this account fail? To answer this question we have to consider what are, according to Etchemendy, the properties essentially attached to the proper concept of LC:
(1) it is necessary that the conclusion is true if the premises are true,
(2) we can establish the conclusion from the premises alone,
(3) the information in the premises warrants the information claimed by the conclusion ([2008] p. 265).
The fundamental point is that the properties of LC involves some kind of justification of the conclusion from the premises: a logically valid argument must be such that the truth of the premises *justifies* the truth of the conclusion. Justification cannot be reduced to plain truth-preservation under all the possible permutations of objects ([1990] p. 93). Tarski's account equates LC to a universal fact: there is no sequence of objects that makes the premises true and the conclusion false. According to Etchemendy, this is just a substantive fact, i.e. a fact about the collection of sequence of objects at our disposal, and there

is no reason to think that it shows any modal, epistemic or informational relation between the premises and the conclusion (*ib*.).

4. Etchmendy's Oversimplification

Etchemendy's assumptions oversimplify the task. I will recall some historical facts to show that his assumptions are far from be obvious. In particular: (1) it is not true that LC has always been the most central concept of logic, (2) it is not possible to say that there is only one proper intuitive concept of LC, since we find several different characterizations of LC and they are strictly dependent from many other extra-logical ideas.

5. Tarski's Aim

Before we deal with the outlined issues, it is worthwhile to be precise about what was Tarski's aim in [1936]: contrary to what Etchemendy writes, Tarski did not try to give a precise account of the proper concept of LC. He only tried to give a workable definition of a certain way to understand the concept of logical consequence. His aim was a methodological one, not a prescriptive one.

Expressions like "the common concept" of LC or "the proper concept" of LC that we find in Tarski's paper mislead Etchemendy. Tarski simply tried to precisely characterize a way in which the concept of LC can be understood: the way in which it was used in the axiomatic study of mathematical theories . Tarski explicitly denied that his aim was to grasp the right or indeed the only possible concept of LC, since he believed that there is no right concept of LC. The only legitimate task is to make this concept precise in one of the possible ways that one can follow, then to proceed to a systematic study of its properties and of its relations with other concepts ([1944] p. 355) and, finally, to develop a further inquiry into the field of the methodology of deductive sciences ([1995] p. 160).

6. A Manifold Concept

6.1 Aristotle

LC was not a central concept in Aristotle's analysis of inference, which is essentially the theory of syllogism: syllogism is only a peculiar argument (two premises and a conclusion, only categorical sentences, three terms, the extremes and the middle term). Aristotle does not take interest in logically valid argument *per se* since his main interest is to explain how to provide scientific arguments. He believes that every fact can be described by a categorical sentence, i.e. by a sentence that predicates a term of another (*Int* 5), then he deals with the explanation of how we can justify an assertion about the world, i.e. a categorical sentence. He finds that to justify a sentence (the conclusion) expressing a predication between two terms we need two *other* sentences (the premises) expressing predication between two terms in such a way that every premise contains exactly one terms of the conclusion (the extremes) and that the premises share exactly one term (the middle term) that is not contained in the conclusion (*APr* I 29). Aristotle's logical investigation is influenced by his notion of truth and by his ontology (facts are described by categorical sentences) and by the role he attributes to logic (how to provide scientific arguments). The same will hold in most part of traditional logic.

6.2 Bolzano

Bolzano's analysis of inference focus on the objective connections holding among certain objective entities (the propositions in themselves), uniquely composed by ideas in themselves. Since these entities and their mutual connections are objective, he tries to describe them without referring to any inferential practice. Bolzano divides the ideas into two groups: the fixed one and the variable ones. Let V be the set of all the variable ideas and let Γ and Δ be sets of propositions. Δ is compatible with Γ with respect to V iff there is at least a permissible substitution of the representations in V that makes all the propositions in Γ and Δ true at the same time. Δ is deducible from Γ with respect to V iff they are compatible and every permissible substitution of the ideas in V that makes the propositions in Γ true makes the propositions in Δ true as well.

A particular form of deducibility (strict deducibility) is the one that holds between a set Γ and exactly one proposition φ when {φ} is deducible from Γ with respect to V and there is no Γ' such that Γ' is a proper subset of Γ and {φ} is deducible from Γ' with respect to V (I will call this property: non-redundancy). In ML, Bolzano defines LC as a special case of the relation of strict deducibility , since Bolzano thinks that, essentially, the logically valid arguments are syllogisms, in which there is a singular conclusion and no idle

premises: φ is a LC of Γ iff φ is strictly deducible from Γ with respect to V and V is the set of the logical ideas.
Aristotle could not have stated this definition since he did not consider propositions and their parts (specifically terms) as objective entity living in a separate world. Bolzano, then, imposes the requisite of compatibility and of non-redundancy of the premises since, just as Aristotle, he thinks that logic should also be a tool for obtaining new knowledge from the premises and it should be helpful to give a systematic form to scientific disciplines.

6.3 Frege

Also Frege does not study LC *per se*. His aims are to give a formal deductive system to reconstruct every valid arithmetical proofs avoiding deductive gaps and to show that arithmetic is logic. His main interest, then, is in defining what a gapfree inference is. Consequently, he thinks of logic as a calculus whose prevalent interest is rigor and so, contrary to Aristotle and to Bolzano, he does not study only specific inferential forms (i.e. syllogistic figures) and he think that conditions like compatibility or avoidance of idle premises are unessential.

6.4 Following Rules and Formal Semantics Characterization

These historical hints show that Etchemendy cannot simply take for granted that LC must be characterized according to the properties he suggested. We cannot simply speak of the concept of logical consequence. There is a common useful practice in which we refer to LC but its relevance and its general characterization depend on the philosophical ideas and the scientific aims of the different approaches. I think that in the history of logic we find two main characterizations of LC, represented above by Bolzano and Frege : respectively, (1) a formal and semantic characterization and (2) an epistemic and normative characterization.

By (2), I mean a characterization of LC that underlies how you can get the conclusion from the premises (epistemic characterization) and that explain why you can get the conclusion from the premises (normative characterization). The concept of LC is bounded to the concept of following rules. These rules are valid ways of reasoning, so it is contrary to reason to hold the premises and not to accept the conclusion. From this point of view, logically valid arguments must be used both to enrich our knowledge (reasoning takes us from the knowledge of the premises to the new knowledge of the conclusion) and to be sure that the conclusion must be true if the premises are true (reasoning compels us to accept the conclusion if we accept the premises).

By (1), I mean a characterization that equates logically valid arguments to arguments that preserve truth only for their logical form. The fact that the conclusion must be true if the premises are true is explained in terms of generality of truth-preservation where this generality is considered a logical one: the cases taken into account are aimed to represents all the logically possible cases. This characterization emphasizes the fact that the relation of LC is objective and not dependent by rules of reasoning. The fact that logically valid arguments can be used to enrich our knowledge or to explain why something follows from something else is not the central element here, since the emphasis is on the fact that a sentence can follow from certain premises even if there is no way to justify this claim providing a proof (this is particularly salient in the case of the axiomatic study of mathematical theories).

This two kinds of characterization, whose motivations are different, can nevertheless determine the same set of logically valid arguments, but, obviously, this is not always the case (e.g. in second-order logic no inferential practice can determine all the arguments that are logically valid from a semantic point of view). An interesting point is, then, to revise the common understanding of the completeness theorem: a deducibility relation coincides with the relation of logical consequence. This understanding relies on the assumption that the proper description of the relation of logical consequence is the semantic one. The previous analysis shows that, more accurately, we should say that the completeness theorem shows that two different views by which we can describe the concept of logical consequence extensionally coincide (even if their motivation remains different and both legitimate).

Etchemendy did not recognise this plurality in the history of logic and he did not provide any argument to sustain his requirements. This plurality supports the claim that we have a practice in which we use the concept of LC, but we do not have any conclusive intuitions to characterize it in a unique way and, moreover, when we try to characterize it, we build our definition in accordance to our metaphysical and epistemological assumptions and to our legitimately different scientific aims.

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