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Between a bottom-up approach to mathematics and constructivism

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ABSTRACT. The aim of this contribution is to show the compatibility of an embodied and bottom-up approach to mathematics, as developed by Carlo Cellucci, with a constructivist epistemological framework and with a non-adaptationist view of evolution. This different perspectives are shown to be connected with an antirealist stance in philosophy of science.

1. Naturalism and the bottom-up approach to mathematics

The main problem for Naturalism is, perhaps, to account for what mathematics is in a naturalistic way[1], given the crucial role mathematics plays in our scientific theories[2]. If our knowledge depends on mathematics and we want to avoid the Platonistic claim that accepting Scientific Realism (SR) implies accepting mathematical realism[3], we have to look at mathematics as deeply re-

[1] Van Kerkhove (2006), p. 15.
[2] Sober (1993), p. 35.
[3] Colyvan (2001).

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lated to our cognitive and organic structure[4], in order to give an account of our knowledge which does not imply dualism of any sort.

A bottom-up approach to mathematics is an attempt to follow this line of thought[5]. The problem that this kind of approach entails is that is difficult to show that it is compatible with the claims of SR, and so even with those of Naturalism, claims that instead it should support. In fact, a bottom-up approach to mathematics can account for our capability to reach true (or approximately true) knowledge, only if it embeds some realistic assumptions in his presuppositions[6]. But this is quite circular, in such a way a scientific realist should not appreciate.

Here, on the contrary, I'll try to show that the bottom-up approach to mathematics developed by Cellucci[7] is compatible with a constructivist epistemological framework, and in particular with the view Diettrich calls "Constructivist Evolutionary Epistemology" (CEE)[8]. The key elements to stress this compatibility are identified in the rejection of the concept of truth, and in the way the process of knowledge is described.

2. Cellucci in a nutshell

Cellucci's view of knowledge, mathematics and logic is based on three major theoretical claims:

1) Mathematics, as logic or reasoning, is deeply rooted in our biological make-up[9]. This key move forces us to investigate epistemological problems which are normally debated in philosophy of biology rather than in philosophy of mathematics.

2) Cellucci shifts from the concept of truth to that of plausibility[10]. This is deeply connected with his "biological perspective": if our knowledge depends on our natural structure, and to account for this we have to look at evolutionary theory, we should conclude that our knowledge cannot be seen as the product of a true statements producer.

[4] De Cruz (2006).
[5] Cellucci (forthcoming); De Cruz (2006).
[6] See below, section 4.
[7] Cellucci (2008); Cellucci (forthcoming).
[8] Diettrich (1994).
[9] Cellucci (2008), p. 206.
[10] Cellucci (2008), p. 89; p. 177.

3) The method which best describes our knowledge process is, according to Cellucci's view, what he calls the analytic method[11]; from hypothesis to hypothesis, being constrained to account for data related to the problem we are trying to solve, to find a sufficient solution, and so on, *ad infinitum*.

What is important here to underline is that in this approach the search to find what mathematics really is, and so what human knowledge really is, is connected to the investigation of evolutionary processes and of our cognitive structures, and that these two latter are mutually connected. At this point, an epistemological problem of circularity arises, given that the study of evolution and cognition is a scientific effort, and so involves human cognition and mathematics. This sort of circularity is a problem for a realist perspective, because it denies the possibility to reach a direct (or even indirect) comparison with reality in order to state the truth of the knowledge produced. But it is not a problem for a constructivist perspective.

3. Antirealism in philosophy of science and in mathematics

One of the strongest argument for SR is the No Miracle Argument (NMA)[12], which can be seen as an Induction to the Best Explanation (IBE)[13]. In philosophy of mathematics, Platonists capitalize on this kind of argument, using indispensability arguments[14] to support their claims:

> Scientific realists, (…), are generally committed to inference to the best explanation (…). Indispensability arguments about mathematics urge scientific realists to place mathematical entities in the same ontological boat as (other) theoretical entities. That is, it invites them to embrace Platonism.[15]

Naturalists try to resist to Platonism naturalizing mathematics. I'll try to show that if we try to naturalize mathematics, is difficult to maintain realist claims. If we want to root our mathematical abilities in our cognitive and bodily structures, we should renounce to conceive our knowledge as guaranteed. Otherwise, if we want to guarantee our knowledge through mathematics, we are ex-

[11] Cellucci (2008), chapter 14.
[12] Psillos (1999), p. 68.
[13] Frost-Arnold (2010), p. 45.
[14] Psillos (2010), p. 947.
[15] Colyvan (2001), p. 8.

posed to Platonistic, or at least Pythagorean[16], claims. It is possible to root mathematics in our biological constitution developing an account of mathematics as that proposed by Cellucci and support it with a view of the evolutionary process distinct from adaptationism[17].

4. Antirealism in philosophy of biology

A realist stance in philosophy of biology (broadly intended to include neurosciences and problems related to philosophy of mind) can be shown to be deeply connected with adaptationist views of evolution and (some) representationalist views of cognition[18]. In order to naturalize human knowledge and avoid Platonism, in fact, the attempt of naturalizing our cognitive ability through evolution is deeply related with the complete naturalization of human mind.

Adaptationism. If what is crucial in evolution is selection, and selection is intended as a variation in fitness, there are some strong assumptions and ontological commitments we are forced to accept in order to support such a view. For example, to list just some of the most debated issues, there is a clear assumption about what causation is and on how to describe it[19], on what a trait is and on how to individuate it[20], on what fitness is and on how to measure it[21], on what a gene is and on how it is linked to a trait[22]. If these models are intended as probabilistic, they (probably) imply a realist perspective on probability[23]. Moreover, mathematical models, and sometimes mathematical explanations, can be used in evolutionary thinking[24]. This brings the circularity problem mentioned above: if biology is used to naturalize mathematics, but biological scientific theories indispensably use mathematics in their models and explanations, and mathematics is seen as a crucial element to reach guaranteed knowledge, the only way to solve this puzzle for a naturalist is to see adaptation as a truth-encoding process. That is, the relation between organism and environment, should be seen as guided by a process which is able to en-

[16] Steiner (1998).
[17] Sober, Orzack (2001).
[18] Stewart (2001), p. 110.
[19] Okasha (2009).
[20] Lewens (2009).
[21] Lewens (2010).
[22] Moss (2001); Lewens (2009).
[23] Sober (2010).
[24] Baker (2009).

code in us true (or approximately true) knowledge about the world[25], or to give us a faculty which is able to reach some true (or approximately true) knowledge about the world in dealing with it. This relation is *adaptation* and this process is *selection*. So, adaptationist view seems to be the most adequate account for evolutionary process in order to maintain a realist perspective.

But if adaptationism can solve the circularity related to the use of biology to explain mathematics and the use of mathematics to build up biological theories, because the truth of mathematics is guaranteed by selection, and so the truth of this human product is based on true features of the world, adaptationism itself can hardly guarantee the crucial assumption in the above argumentation: the fact that the relation between organisms and their environment can be seen as a sort of transfer of true knowledge, managed by selection, which is able to guarantee the product of our knowledge. This assumption can only be sustained by an IBE. This is a "success-to-truth" inference[26], for which the survival of an organism implies his true (or approximately true) knowledge of its environment. But this sort of inference is exactly the inference at the core of SR. And this kind of inference is just what an antirealist would resist to[27]. So, adaptationism is crucially based on some realistic assumptions, and can be hardly used to support SR, and to naturalize mathematics in order to justify its importance for our scientific theories.

Representationalism. The same kind of relation can be shown to be at the core of one of the most influential attempt to naturalize cognition[28]. The relation between the subject and the world is seen in the realist tradition as an attempt to mirror in a true (or approximately true) way the world's features[29]. This kind of ability of our bodies and brains should be guaranteed by evolution. But the way of studying and conceptualizing our cognition is based on mathematics and scientific theories and devices. Again, adaptationism is invoked to justifies this evolved human ability of truly knowing the world, selection is seen as the process which has build up this trait, and the survival of the organisms bearing such trait is seen as the proof of its ability to produce true knowledge of the world. Even here the core assumption is that useful and successful knowledge can only be true knowledge, and that true knowledge can only be based on some sort of correspondence with the world. As the truth of scientific theories is inferred from their success, in this kind of adaptive

[25] Dehaene, Brannon (2010), p. 517.
[26] Saatsi, Vickers (2011).
[27] Frost-Arnold (2010).
[28] Stewart (2001).
[29] Musgrave (1996), p. 23.

thinking human ability to produce true knowledge is inferred from our survival. So, even our cognitive processes cannot guarantee our knowledge, unless we look at them from a perspective which is already realist in his fundamental assumptions. Difficulties arise when trying to account for our capability to mirror the world, because of the absence of an adequate truth criterion[30].

The price to pay to maintain the certainty of our knowledge is to support a sort of view which is deeply related with a form of more or less explicit dualism[31]. To assure that the organism-environment relation and that the subject-world relation can entail true knowledge, realist explanations in biology use the concept of information[32]. This leads to a quite paradoxical situation: to account for our ability in producing true knowledge of the world, we have to posit that what really counts in the crucial processes for our knowledge (i.e. evolution and cognition), is a non material feature of the material world we are trying to truly describe and explain through scientific theories[33].

There is no sufficient room here to explore this issue, but this move just shifts the problem from the search to account for mathematics and its role in our knowledge, to the search to define what information is and its relation with our cognitive structures and with mathematics.

For a naturalist, trying to resist to Platonism adopting an informational view of evolution and cognition, could mean not to be able to resist to Pythagoreanism[34], which probably cannot be said to be dualist, but surely is anti-naturalist[35].

5. Diettrich in a nutshell

The epistemological framework Diettrich elaborates is an attempt to give reason for evolution and cognition without accepting realist claims, avoiding to refer to some concept of truth. This is reached eliminating any reference to the concept of reality, which entails a way to describe reality objectively and to verify the correspondence of our knowledge to such reality[36].

[30] Cellucci (2008), p. 85.
[31] Oyama (2010), p. 413.
[32] Oyama (2010), p. 409.
[33] Oyama (2010), p. 410.
[34] Steiner (1998).
[35] Steiner (1998), p. 54.
[36] Diettrich (1994).

CEE looks at the two main elements for a naturalization of human knowledge, i.e. evolution and cognition, in such a way that is compatible with Cellucci's view of mathematics. In fact, Diettrich, as Cellucci, deeply roots our knowledge and cognition in our biological structures, but rejects the adaptationist view of evolutionary process. Diettrich tries to describe the organism-environment relation and the subject-world relation without referring to an objectivist description of reality, but accepting the sort of circularity our cognitive closed processes and the structure of our nervous system seem to suggest[37]. This attempt implies the renounce to the certainty of our knowledge.

Diettrich's position allows us to support the first and crucial theoretical claim of Cellucci's vision listed above (the relevance of biological evolution for mathematics), without being forced to accept an adaptationist, an therefore realist, account of evolution, which couldn't be coherent with the Cellucci's rejection of the concept of truth.

The "reality free" representations Diettrich refers to, seems to be obtainable from two of the principal points in Cellucci's view: the notion of plausibility and the infinite process of moving from hypothesis to other hypotheses, as described by the analytic method. As in Cellucci these points are not just a description of mathematical thinking, but can even be used to describe natural processes linking organisms and environments, so in Diettrich evolutionary processes are structured in a way which is similar to that of cognition, but in both cases the knowledge involved cannot be said to be true, even if is possible to account for the survival of organisms and for the success of theories[38]: what can account, at least partially, for the applicability of mathematics is the common origin that the cognitive structures underpinning mathematics share with our perceptive and other cognitive structures, and the fact that some of the structures involved in mathematical reasoning are the same involved in other cognitive processes connected with the production of knowledge[39].

Such a phylogenetic common origin doesn't entail that these cognitive structures truly represent or incorporate in us some features of the world: it is sufficient that were sufficient to contribute (among other factors), or that at least didn't impede, the survival of organisms, and for this are not required to be true, neither in Cellucci's, nor in Diettrich's proposal[40].

[37] Diettrich (1994), p. 76.

[38] Cellucci (2008), pp. 207-208.

[39] Diettrich (1994), p. 88.

[40] This is exactly the reason why Psillos (1999), p. 74, rejects a Darwinian account for scientific theories: "There is no warrant that they will be successful in the future".

6. Conclusions

We can see the positions analyzed above as belonging to the category of "epistemic antirealism"[41], which doesn't immediately imply Relativism, even if rejects the mainstream reactions towards Relativism, all relating to some concept of truth. This just to briefly remark the mutual implications occurring between such an account of mathematics, a non-adaptationist stance in philosophy of biology and an antirealist perspective in philosophy of science.

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[41] Coliva (2009), p. 184.

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