The onto-epistemological background of Archimedes' máthema

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1. Mathematics from the late nineteenth century to be early twentieth century and the discovery of Archimedes' so called-mechanical method.
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ABSTRACT. Admirable discovery of Archimedes' so-called "mechanical method"(Greek text: *tropos tes theorias dià ton mekhanikon*) in the early twentieth century raises some important historical-philological and philosophical- epistemological questions which are still topical. It is argued about observing (*theorein*), discovering (*euriskein*), showing (*phainesthai*),proving (*deiknunai, apodeiknunai, epi-deiknunai*),giving faith (*pisteuein*) to scientific propositions and filling together (*sum-pleroun*) in Archimedes' onto-epistemology (*philosophia*). In his bibliographical note in addition to the Italian edition of *Archimedes* by E.J. Dijksterhuis, W.R.Knor puts forward the idea of a total silence of Archimedes in philosophical questions. We believe that Archimedes' letter to Eratosthenes has a strong philosophical content, as Archimedes turns to one whom he claims to be "excellent teacher of philosophy", talking, of course, like a philosopher. We have been convinced to rethink Archimedes' onto-epistemological background by a careful and simultaneous reading of the Greek text and non-unique translations of various scholars of terms, as *theorein* and *phainesthai*, by the questionable interpretation of the keyterm of the so-called mechanical

method , as *sections-weights* (tomaì-barea = toma…-b£rea), by the forced translations of the expression, *khorìs apodeìkseos*, by the negligence disregarding the meaning of the Archimedean term scientific "faith (p*istis*)". We think that a Platonic and formalistic philosophy of mathematical research has had a certain weight on the standard interpretation of his method since, between the late nineteenth and early twentieth, starting from Dedekind, has been broken the link which in Archimedes linked mathematics to physics, numbers to magnitudes, mathematics to philosophy. A monistic interpretation of Archimedes' **máthema** is set against the dualistic standard interpretation of Archimedes' epistemology, divided between research and prove, intuition and rigor, provisional and apparent knowledge of mechanical kind and final and true knowledge of geometric one, and of his ontology, divided between the world of pure thoughts and the corporeal things, narrated by Plutarch, inspired by Plato's philosophy. Such **máthema** is be considered as a thing at same time geometric, physic and philosophical, as intend on reaching a content of truth, inspired by the Democritean philosophical principles, since, among the very few, Archimedes mentions only Democritus and never Plato and Aristotle. Archimedes is inserted inside the tradition of Pythagoras, Parmenides, Archytas, Eudoxus, Democritus' thought, which is Italic and not Ionic, like Plato and Aristotle's one , expressing Alexandria's dominant culture , with which he compares himself. We try to demonstrate everything through a careful examination of the Archimedean text, even of the Greek one. In addition to the classic texts of literature on Archimedes, we also refers to various articles and texts on the history of calculus.

1. Mathematics from the late nineteenth century to be early twentieth century and the discovery of Archimedes' so called-mechanical method.

The admirable discovery of Archimedes' mechanical method in the early twentieth century on the part of the Danish philologist Heiberg with the *Letter to Eratosthenes* could not ignore the new philosophical-epistemological trend that was spreading from the late nineteenth and early twentieth century in the conception of mathematics.
Separating the numbers from the magnitudes , and consequently, mathematics from physics, it was thought that, in such a way, it would have been possible

to free calculus from its unclear, intuitive, geometric basis and to instead base it on rigorous axiomatic- arithmetic foundations.

Dedekind wrote that since " *I made use of considerations of geometrical orders in my differential calculus lessons and felt the lack of a truly scientific basis of arithmetic, my sense of dissatisfaction was such that I firmly decided to reflect upon the matter until I found a purely arithmetic basis absolutely rigorous with the principles of analysis.*"(1)

In this way a path was opened on the conception of the object mathematics, even more formalized and more and more unlinked to its "**knowledgeable content**" reduced to just a pure symbol or sign, or to mere relations or games between these.

Thus, if the mathematical historian Boyer, in his book "*The concepts of the calculus*",1939, could write that, at such a point ,the Pythagorean dictum was realized: **All is number** ! (2) ,continuing forward in his text, he can state, categorically , that which is " *the nature of mathematics, as accepted at the present time. Mathematics is neither a description of nature nor an explanation of its operation; it is not concerned with physical motion or with the metaphysical generation of quantities. It is merely the symbolic logic of possible relations, and as such is concerned with neither approximate nor absolute truth, but only with hypothetical truth. That is, mathematic determines what conclusions will follow logically from given premises*.(3)

If, within this historical framework falls the discovery of the "*mechanical tropos*", of Archimedes, there is, on the other hand, an iconographical image in the scientific community which supports Archimedes' image throughout the previous centuries.

Because it was believed that his geometrical demonstrations weren't discoveries, but only demonstrations of things already discovered, the myth had been spread throughout the centuries of an Archimedes who must have used a "**secret way**" while discovering his theorems.(4)

If this epistemological myth supports Archimedes' image, a not less strong myth, but, this time having an ontological nature, accompanies Archimedes' image from ancient times, imposed by the neo-Platonic tradition, about a platonic type, clear cut dualism in his conception of the "*mathematical thing*".

For Archimedes, on one side, stood the world of geometrical purity of pure thoughts (*ton noenton* , according to Plutarch) of which mathematical science occupies itself, while on the other side the physical world ,of all bodily things (*ta somaticà*) of which non-liberal knowledge occupies itself , such as mechanics of which principles and beings weren't confused with geometry.

Therefore, here is , following this, what Plutarch writes concerning this purpose in one of his interesting testimonies:

Eudoxus and Archytas had been the first originators of this far-famed and highly-prized art of mechanics, which they employed as an elegant illustration of geometrical truths, and as means of sustaining experimentally, to the satisfaction of the senses, conclusions too intricate for proof by words and diagrams. As, for example, to solve the problem, so often required in constructing geometrical figures, given the two extremes, to find the two mean lines of a proportion, both these mathematicians had recourse to the aid of instruments, adapting to their purpose certain curves and sections of lines. But what with Plato's indignation at it, and his invectives against it as the mere corruption and annihilation of the one good of geometry, which was thus shamefully turning its back upon the unembodied objects of pure intelligence to recur to sensation, and to ask help (not to be obtained without base supervisions and depravation) from matter; so it was that mechanics came to be separated from geometry, and, repudiated and neglected by philosophers, took its place as a military art.(5)

Notwithstanding, we went out, at the end of the nineteenth century and beginning of the twentieth century , from a chaotic and impetuous development of calculus, having taken place since the sixth, seventh and mid-nineteenth century in which mathematical and physics research had been deeply stirred into the various Newton, Euler, Laplace, D'Alembert, etc.

The rigorous placement of this new knowledge has been neglected, as it was written, because into his were leaving beings within form of ghosts, which were the infinitesimals and the indivisibles. There was a need to sort out, to regain rigor within a framework of light-dark elements that the new science of calculus had developed over the recent centuries.

Archimedes, during those impetuous centuries, was assumed to being an **icon of rigor** for the systematic use of the so-called "*method of exhaustion*" against research and presumed unscrupulous and unprejudiced demonstrations.

However, Archimedes remains the master of rigor, but his presumed geometrical purity of the Platonic type becomes purified to an even more pure purity, the arithmetic one, as we have already seen with Dedekind.

Nevertheless, the purifications for evermore pure beings will continue .

The discovery of Archimedes' mechanical tropos if on one part puts into crisis the traditional myth of an Archimedes icon of rigor, of the use of the geometrical way through the method of exhaustion, on the other hand, it seems to initiate another myth of another Archimedes, broadminded re-

searcher, who, apart from the geometrical tropos, in his research, relied on another tropos, that mechanical one of which was first noted in the finding of the letter to Eratosthenes.

2. The new historical-epistemological environment influences the interpretation of the new Archimedes and the translations of his texts.

The new cultural trend, with its onto-epistemological status, Platonic and formalistic, could not tolerate that the great Archimedes, in his theory of the "mathematical thing" would have been able to mix the "mechanical thing" with the "geometrical thing", neither the less that he could have mixed it with the "philosophical thing", since the new mathematics, being interested in pure linguistic form, seemed to take no interest in its "meaning of truth" ,or at the more admitted a his purely ideal existence of a Platonic type.

   Once again, it is attempted to keep Archimedes's presumed Platonism alive by creating **a new onto-epistemological myth of still a dualistic type**, therefore, Archimedes' geometrical thing   is interpreted   as pure "geometrical thing" , seat of a totally ideal truth, opposed to the appearing "mechanical thing" , which anticipates the notion of the geometrical truth, while, at the same time, the object of knowledge, **the máthema** is separated between the object of research and the object of demonstration, , the object of intuition and the object of rigor, and between the object of apparent mechanical knowledge, and the object of true geometrical knowledge.

  To quote some historian, let's first have a look at what Boyer writes:

   Archimedes of Syracuse displayed two natures, for he tempered the strong transcendental(?) imagination of Plato with the meticulously correct procedure of Euclid (6)

  and then in his monograph Dijksterhuis:

   "*Archimedes is not prepared to recognize the results obtained with twofold method ("barycentric   method" and "method of indivisibles")as actually proved conclusions* (7)

"*The mathematical deficiency is exclusively a consequence of the use of indivisibles*"(8)

 " *In this treatise Quadrature of the Parabola which constitutes a official publication satisfying all requirements of exactness, he proves the insight gained in Prop. 1 on the area of any segment of an orthotome   once more by means of statical considerations ,but this time without indivisibles* (9)

"*The method has revealed to us that the indivisibles had only been banished fro he published treatises ,but that in the workshop of the producing mathematician thy held undiminished sway*" (10)

It is clear that this dualistic reading of Archimedean texts influenced the new translations of the new texts, thanks to the new, historical-cultural environment of the early twentieth century , mentioned from us and of the always present and renewed myth of an Archimedes, who was not only Platonic but also possibly Aristotelian (11).

3. The various translations of the new Archimedean text.

We now come to some critical aspects. From the simultaneous readings of Greek text, in general and in particular, of the *Letter to Eratosthenes* and from the various translations this is what is noticed.

The different translators or do not translate in the same way, without respecting a precise scientific criteria, key- words from the text (not noticing, among other things, the reader that the word" *method*" translates two Greek words, one more generic than the other and only used once, *éphodos*, which gives the title to the *Letter to Eratosthenes,* while the other is used appropriately by Archimedes and is "*tropos tes theorias dià ton mekhanikon,*" which we can translate "**the way of observing by mechanical beings**" ,which is repeated several times and acts as a framework during the demonstrations of the mechanical type - *theoréitai dià tou tropou toutou* = **one observes by this way**) or they rely on interpolations, which are , in our opinion, not very convincing, or they commit some real and often serious omissions, forcings of the literal text as also of its meanings.

 Not to say that they overlook terms that could help to complete a proper interpretation of Archimedes' onto-epistemological statute .

The translation of the same term of "*tropos tes theorias dià ton mekhanikon,*",as "**theorein**" in various ways, using different inflections, both verbal and nominal on the part of the same translator, and also on the part of several translators ,certainly doesn't help the comprehension of Archimedes' onto-epistemology, but does, however, inflict confusion or even the forced interpretation which can result from it.

T.L.Heath translates it with *to investigate*, generally, not always, but then translates "**theoria**" with *inquirry*. (12)

E.J.Djksterhuis in his English text *Archimedes* ,Copenhagen,1956 ,translates it with *to recognize*, but then translates "**theoria**" with *investigation*. (13)

C.Mugler translates it with *d'aborder*, *examiner*, but then translate "**theoria**" again with *étude*, a generic term. (14)

P.Ver Eecke translates it with *se presenter,veni à bout,*,to then translate "**theoria**" with *investigation*. (15)

Frajese translates it with *considerare, vedere, esaminare,trattare*, to then translate "**theoria**" first with *teoria*, but also with *ricerca*(16)

In fact, there is a forcing when *euriskein* is referred to, in what we have called an interpretation of the dualistic type, only at the time of the mechanical discovery, not recognizing that there's also in Archimedes a *euriskein* of a geometrical type, better yet a dialect between discovery and demonstration in the two tropoi, mechanical and geometrical.

It is simply better to examine the text to understand that Archimedes doesn't **distinguish mechanical research from "true" demonstration, but <mechanical demonstration> from <geometrical demonstration**>

So much as Archimedes, at the end of his *Letter to Eratosthenes* ,writes about the two theorems, of which he'd only sent him the enounced ones, inviting him **to discover** the demonstrations, that he is now able to, after having **observed** them with the same mechanical tropos, thanks to the **appearing** for the first time of the theorem on the surface of a section of a rectangle cone, to also give the geometrical demonstrations(*tas geometrikas apodeikseis)* that he'd discovered.

Archimedes concludes his **Letter to Eratosthenes**:

*"Thus, we before write the first of the theorems, appeared (**phanén**) by means of mechanical beings that any segment of a section of a right –angled cone is four –thirds of the triangle which has the same base and equal height , after this each one being **observed** by the same **way** (tropos); and so at the end of the book we write the **geometrical demonstrations** (tas geometrikas apodeikseis) of those theorems, of which we had previously sent the enunciations."* (17)

From the beginning of the letter, it appears clear that, if there were a discovery of the mechanical tropos , there'd also be a discovery of geometrical tropos, which is what Archimedes invites Eratosthenes to do, but then done by Archimedes himself as he writes in the letter.

"*I sent you on a former occasion some of the theorems discovered(*with mechanical tropos; ours) by me, merely writing out the enunciations and inviting *you **to discovery the**(**geometrical;ours) proofs**, which at the moment I did not give*" (18)

Phainesthai, another key term of the "*tropos tes theorias dià ton mekhanicon*", Heath translates it once *to become clear*, then once again *to become known*, and once again *indication* when a name is given (*émphasin*) (18).
Not to speak of other translators of the term.

Thus, it is attempted to oppose "**an apparent being**" of mechanics to "**a true being**" of geometry.

Therefore Dijksterhuis can always write with regards to Archimedes as well:

"*In mathematics a discovery is one thing and a proof quite another, and that the method by which the reader is convinced of the truth of a theorem in many cases is quite different from the way in which it was first found*" (19)
Which it is not always true, more so in the case of Archimedes.

In fact, that is how the Archimedean text becomes disturbed, for which **what is *true* (alethès**) is referred to as "**the observed thing**" which **appears** (**phainesthai**) thanks to *tropos tes theorias dià ton mekhanicon*, but hasn't yet been demonstrated geometrically.

In fact, Archimedes writes in the following passage of the prop.1 of the so-called *Method* : *Now the fact here stated is not actually demonstrated(**apodédeiktai**) by the argument used; but that argument has given a sort of indication(**émphasin**) that the conclusion is true (**alethès**)$_1$. Seeing then that the theorem is not demonstrated* (by geometrical beings; our*s) , but at the same time suspecting that the conclusion is true(**alethès**)$_2$,we shall have recourse* (**tàksomen**) *to the geometrical demonstration* (**ten geometrouménen apodeiksin**) *which I myself discovered and have already published"(*20)

Another time Archimedes refers the false(**pseudés**) to the proposition or better yet to the enunciation, *protasis*, before this was **observed in its truth or falsity**; this is what is taken from his letter to Dositheus on *Spirals*.

"*Of all the propositions just enumerated Heracleides brought you the proofs. The proposition stated next after these was wrong(**pseudés**), viz. that, if a sphere be cult by a plane into unequal parts the greater segment will have to the less duplicate ratio of that which the greater surface has to the less. That this is wrong(**pseudos**) is obvious* (from the demonstrations given **geometrically**, but have appeared before, as with the mentioned case, **by mechanical way** ; ours; **On the Sphere and Cylinder,II,8**) *by what I sent you before; for it included this proposition: If a sphere be cut into unequal part by a plane at right angles to any diameter in the sphere, the greater segment of the surface will have to the less the same ratio as the greater segment of the diameter has to the ,while the greater segment f the sphere has less a ratio less than the*

duplicate ratio of that which the greater surface has to the less, but greater than the sesquialterate of that ratio"(21)

There will be a clearer focus later , when a more organic discussion will be made of all Archimedes' onto-epistemology or philosophy, through these sporadic expressions, in Archimedes' proportional logic, on his dialect about true or false. Concerning the questionable interpolations of the translators I refer to the case of **korìs apodeikseos**, literally *"without demonstration"*, which Frajese and Rufini (see Enrico Rufini, *Il metodo di Archimede*, Feltrinelli, Milano,1961,p.103) translate instead, *"senza una vera dimostrazione"* "and Heath and Dijkstehruis *"without an actual demonstration"*. While, remaining in the context, rather than *"without demonstration"*, the adjective "**geometrical"** should be added.

To force in the other sense ,significant expressions are omitted from the translated text, as with the case of the theorem 3 of the Method, about the volume of a spheroid, which ends with the Greek phrase "*oper édei deikthenai*" (**as should have been demonstrated)**, but is reported neither by Frajese nor by Dijkstehruis.

Commonplace errors (!?) of translations even are made, in Frajese, for the sake of denying any demonstrative value to the "*tropos tes theorias dià ton mekhanicon,* when a plural, in the original text, is exchanged with a singular in the translated text, as with the case, in the letter to Dositheus of 1st book of *The Sphere and the Cylinder* , of *demonstratio* translated instead of "*demonstrations*"(Greek text, *ton apodeikseon*) about the things that Archimedes had before observed (**teteoremena**), around a section of a rectangular cone (*demonstractionibus adiunctis conscripta* Heiberg rightly translates).

In the same way, no attention is made to the fact that Archimedes calls "*demonstrations*" , at the end of letter to Dositheus in *Quadrature of parabola* that which is obtained around the area of a section of a rectangular cone both by *dià ton meckanikon* and by *dià ton geometrikon* .

In Heath's translation then of the letter to Dositheus on *Quadrature of the Parabola* , in addition to the plural "**disappearing**", the term "**faith or trust**"(*pistis)* disappears, because the text is paraphrased.

Nonetheless, we retain that this has its importance in the interpretation of Archimedes' **onto-epistemology**, an expression we use to interpret and translate his term "*philosophia,*" present only once in the entire Archimedean text , when Archimedes refers to Eratosthenes, quoting him "***a man of zeal and furthermore a man worthy of speaking about philosophy***"(*spoudàion kaì proestòta aksiològos philosophias*) ," **to whom he would like to speak with as philosopher recognizing his competence and passion for philosophy**.

I will say Archimedes, who silenced on the topic of philosophy, as Knorr tries to make infer (22), if one wants to pay attention to what is being said in this significant expression!

This is also used to confirm bad and prejudicial interpretations or to corroborate his presumed Platonism or Aristotelianism underground in his work.

4. A monistic interpretation of Archimedes' onto-epistemology. The problem of the so-called indivisibles.

We enter more ,with his letter , than in his **mathematical workshop** , to stay with Zeuthen's expression, in his **philosophical workshop**.

In fact, being blinded by the presumed Platonism or Aristotelianism of Archimedes, the profound ontological meaning is not captured, in a Democritean key, of another term of his mekhanical tropos (*sum-pleroun*), present and systematically used by Archimedes in his demonstrations of a mechanical way.

Irony of fate! Democritus is the only philosopher mentioned in one of his works by Archimedes, and wrote about mathematical things, while neither Plato nor Aristotle are mentioned by him nor have they written about mathematics, but only witnesses scattered here and there in their writings and very often confused(23), yet they're considered Archimedes' inspirers!

The arguable interpretation of Archimedes' "*koris apodeikseos*" as of the entire context has come to assume that in Archimedes' citations there are references to presumed and hypothetical **indivisibles** of which Democritus would have used in his research, and which were then strictly systematized by Eudoxus ,by geometrical finished   method, while from quoted text, as from other witnesses about Archytas, we learn that themselves have even used in their research   mechanical means. Therefore, it is also supposed that even Democritus, being versatile and universal mind , also made use of mechanical means during his research, considered, among other things, that the whole letter focuses on the two mechanical and geometrical ways, *tropoi,* to treat mathematical things.

The possibility to be considered then is that Archimedes, with his mechanical tropos remains inside a tradition of thought on the mathematical thing or **máthema**, of a unitary nature, or using a more philosophical term, monistic, having its historical roots in Pythagoreanism and in its propagation, the Italic school, according to D. Laertius' diction (24), under the   denomination of which he regroups Pythagoras, Philolaus, Parmenides, Zeno, Archytas, Eudoxus, Democritus, etc. , which opposes to the Ionian school, under the   de-

nomination of which he regroups Plato, Aristotle, and others, which has contrasted and made to stop the other, as can be read in Plutarch, and we inferred from witnesses on Plato and the Neoplatonics.

It was said that Plato would have wanted to burn Democritus' works, and if it hadn't been possible for him to do so, nevertheless he hatched about him a plot of silence, never mentioning him in his writings, though mentioning the names of all the so-called pre-Socratic people. The Neo-platonic Proclus is one of the actors of the conspiracy, if it is true that he doesn't mention Democritus among all the mathematicians who lived before Euclid (25), although Democritus did write about mathematics, and though being mentioned, among the very few, by the great Archimedes.

Thus, in the corners of Archimedes's philosophy, above all, we don't find a indistinct   myth   to the research of the **simplicity**(26), which is unknown what it is, neither a indistinct , generic and divine-like **intuitive method**, of which he relied on , and that it is still unknown what it actually is , but rightly precise principles of onto-epistemology, one of which was the **research of symmetries** in reality (27), which can be said carries with it the sign of the famous principle of Philolaos of Croton, Pythagorean and Italic : *All things known have a number : without a number, it'd be impossible to think about or to know anything"*(28).

Within this principle Archimedes moves guided by a strict and chain-like knowledge of technical, geometrical, mechanical, and arithmetic notions, since Archimedes wrote books, as we learn from his "*Arenarius*", even of arithmetic, besides by a mental discipline of observative, imaginative, and logical-mathematical type, not attributable neither to   Aristotelian empiricism nor nonetheless to the Aristotelian logic.(29) All this can be regrouped within that which is said to be his *tropos tes theorias dia ton mekhanikon*.

We can find two principles of Democritus' philosophy of the full, *to pleres*, and the void, *to kenòn*, in his philosophic-epistemological workshop.

Geometrical figures as a cone, a cylinder, a sphere, a spheroid, or conoid etc, idealized and imagined empty, are filled completely from time to time through circles-sections-weights (*tomaì-barea*), deemed full physical elements, , putting them together *(sumplerothentos,* Archimedean term). Figures in their physical fullness, with unitary density like their elements, carried onto a lever, are in equilibrium, thanks to their weights according to a determined ratio.

In these principles, especially in those , as indivisible magnitudes or bodies (*atoma meghete or atoma somata*) it was said that Archimedes individualized

the weak element, the **vulnus** of his mechanical tropos, such as to make of it just a simple research tool, but not of demonstration .

If in no doubt these anyhow are the two physical principles with which Archimedes constructed his mechanical demonstrations, and these are of Democritean origin, that have nothing to do with Platonic philosophy or with Aristotelian one, **it is necessary to have caution not to read Archimedes with the eyes of Aristotelian critics and of his school of so-called indivisibles or indivisible magnitudes,** as we believe that of these Dijsterhuis implicitly refers to in his cited passage. In the Archimedean text such notions aren't cited, also because the great Archimedes wasn't ashamed of them if at the end of the letter he says he wants to publish his method and confides in the fact that it could be spread among the mathematicians to use in their research, and, also above all, because Archimedes, as already mentioned before, seems to move himself within a tradition of thought, in which the idea of atom, or better yet, **full element,** *to pleres*, had an absolutely **different epistemological meaning.**

In reality, writes S.T. Heath, *they* -parabolic segment and triangle - *are made up of indefinitely narrow strips, but the width – **dx**, we might say - being the same for the elements of the triangle and segment respectively, divides out*.(Heath S. T., *A History of Greek Mathematics* ,Volume II, Dover Publications, New York, 1981,p.557, p.30). And still Boyer writes : *a collection of thin laminae or material strips.*(Boyer, 1939).

Relying on numerous testimonies, Democritus called **full** that which most probably the disciples had simply called atom, indivisible.(30)

According to the Italic tradition of the Parmenides and of Democritus, in fact, the true element is **the thought thing**, not the sensible thing, and it is thanks to that we are able **to observe** (this is the meaning of *theorein* of the *mechanical tropos* of Archimedes) the **physical** thing, in a **real way**.

Aristotle, as an empiricist, confuses **the sensible thing with the thought about thing**, with a physical thing, therefore it is clear that an indivisible magnitude, as a sensible thing, is a contradictory thing, but if it is considered as **theoretical element** (as I think that was the **atom-idea** in Democritus, according to some testimony on him ,and this seems to be in Archimedes, or **the idea of full**), then it escapes the contradiction.

*"Democritus believed that indivisible bodies were principles of the things, but as theoretical elements, **logoi theoretà**, (See Luria S.).*

Atoms or the full elements are indivisible because they are conceived in such a way, without pores **,but, as they are full and fill a physical space, they have parts and are magnitudes**; they are potentially divisible to infini-

ty, which is then the third meaning of "indivisible" attributed from the Aristotelian Simplicius to Democritus.(31)

This explains why Democritus, always relying on witnesses, supports the idea that atoms exist **both small ones and ones as big as the world.**(32).

Only within this onto-epistemological framework could have then originated all modern concepts of physical science of the <material point>, of <rigid body>,etc., incompatible with an empirical philosophy of Aristotelian type.

Thus, full atoms did not only live a secret life of the workshop , as Dijsterhuis supports, but had had a long existence, inside the tradition of Italic thought(33) of which Archimedes continued the tradition, but had been oppressed by the Alexandrian culture under the weight of the Platonic and Aristotelian tradition.

From here the ambiguities of expression, the difficulties in dialogue and with the relations with Alexandrian geometries in the course of his letters. With these geometry had separate from mechanics, and so from philosophy.

In Archimedes' opinion, **the máthema is one**, mechanical, geometrical, and philosophical, since the conclusions resulting from the mechanical demonstrations are true, they have a content of truth, and so a philosophical content, besides the mechanical one and then the geometrical one too, when they're demonstrated in a geometrical way (*geometricòs*), only using lines, surfaces, and solid figures.

Archimedes doesn't boast about Alexandrian surveyors, sending false theorems, which he then belies with true theorems, since these falsify the others, only **after having been observed with the mechanical tropos** , as is the case in the cited theorem, previously seen.

Here we find the internal dialect of true and false, that moves the mechanical demonstrations in relation to the geometrical demonstrations.

If, in the first ones, placing the foundations (*touton dè upokeimenon*, see **On the the Equilibrium of Planes, Book I) the true rises from the bottom, from the postulates, towards above**, the true conclusions, in the second ones **the false descends from above towards below, that is, the premises**, liberally assumed , which Archimedes calls in his "*On the Sphere and Cylinder*", *lambanomena,* and their consequences in the process of geometrical demonstrations, in the case these would conflict with the true propositions of the mechanical way.

Mechanical type knowledge isn't a vague knowledge, nor indefinite and preliminary, a sort of *"some"* knowledge (*gnosin tina* ,see **Appendix),** that then geometrical knowledge renders distinct, clear, and full, but it is the compass which indicates a point of arrival or a direction, a tropos exactly , in its literal

meaning of the word, which guides Archimedes in a **free geometrical re-search made of possible premises and consequences, but compatible with the truth found and demonstrated in a mechanical way.**

Otherwise, it'd be an empty search, a verbalistic turning about among absurd, impossible, non-existent things, like that which happens to those surveyors, who, withdrawing from the mechanical tropos are deceived by thinking they can discover anything, but they only   discover false, impossible things (*euriskein ta adunata,* Greek text), just like what happened to Archimedes himself, when he had wanted to do without.(34)

Archimedes observes (*theorein*) and discovers (*euriskein)* ,and thus demonstrates (*deiknunai)* through the mechanical way; corroborates, discovers, and proves(*apo-deiknuai* ; **apò = ab = as from**) in the geometrical way, "**geometrizing**" the demonstration (=*gheometroumenen apodeiksin*) ; demonstrates, proves and **exposes** through geometrical means   (*epi-diknunai = place in front)* . Concerning this please see what Archimedes writes in his "*Quadrature of the Parabola*", where he says to send Dositheus a "certain geometrical theorem", never observed before him, but has now been observed by him, discovered *dià ton mekhanicon*, and **exposed** *dià ton geometricon* ; **exhibited** translates Heath *epi-deikhtén*").

Who guarantees the truth about the elementary mechanical propositions?
Here we come to the profound nucleus of Italic epistemology, Euclidean and Archimedean: they are **the elementary physical operations that we can achieve and experiment**, which is what Giusti has well put into evidence. (35)

Archimedes widens the numbers of these, from the rule and compass, to the use of the lever in order to allow quadratures and cubatures of curvaceous figures including the physical elementary operations conducted by the lever.

If the **máthema** (perhaps it wasn't necessary to wait for Godel for this) wants to guarantee itself from possible internal contradictions hidden in the system, and to not sterilize itself in an empty verbalism in his process of research and demonstration, it must then move **outwards** from himself, from his linguistic formalism, as also if it wants to have his own content of truth which isn't only of a syntactic nature, but also semantic, having physical and philosophical results, if it, in the end, wants to obtain a cultural content.

Therefore, Archimedes was neither a surveyor, physicist, logistic, nor philosopher, but all of these simultaneously, a great scientist and lover of knowledge, if be said, a real **sofos**, ,a scholarly, in its ancient meaning.

Finally, a last consideration in the investigation of the onto-epistemological meaning of Archimedes' **máthema** .

A scientific result can obtain faith or trust, as we can say, the Archimedean *pistis*, of the scientific community, only if it has undergone an expert check, and is by them check, recognized and accepted. This is what Archimedes requires in his relations with the Alexandrians. He recognizes, in Conon, his main friend and expert in mathematical things, the proper and necessary ability of judgement in his mathematical research. This is an absolutely modern attitude, since it is understood that scientific research isn't just the outcome of observations, experiments and demonstrations ,of **theory**, but of dialogue, a comparing of free research, well, even a **social matter**, which is the question that is divided and compared in the contemporary theoretical epistemological debate**: is science a social institution or a historical trascendental?** (36)

Perhaps Archimedes went even further, since he understood through his relations with the Alexandrians, that the so-called rigor, when of it a fetish is made, is the result more or less of what we now call paradigms, or of philosophies at times declared openly, but at other times not declared and hidden, or of traditions of thought, in competition and conflict against one another, as we prefer to call the whole thing. (37)

Notes and References: with the translation of the main passage in the letter to Eratosthenes, step by step, with the Greek text in parenthesis written in the **Appendix.**

(1) See R. Dedekind, *Stetigkeit und irrational Zahlen* , Gesammelte mathematische Werke,Vol.III, Braunschweig 1932, pp. 315-334.

(2) See C.B. Boyer, *The concepts of calculus* , New York, 1939, p.298

(3) ibidem p.308

(4) Here is what Wallis writes about such a matter reported in *Opere di Archimede*, Utet,Torino,1974 p. 558: "(It seems that Archimedes) *had purposely covered the tracks of his investigation, as if he had buried for posterity the secret of his research method*" and still Torricelli in his passage: *I'd like to believe that the ancient surveyors relied on this method(of the indivisibles) to discover the most difficult theorems, and that afterwards in the demonstration would have preferred another method, either to hide the secrets of art and to not offer to any envious detractors reason for critic.* Torricelli, *Opere* , Utet,Torino,1975,p.381

(5) See Plutarch, *Marcellus' life*, 14, 8-12.

(6) See C.B. Boyer , p.48

(7) See E.J. Dijksterhuis, *Archimedes*, Copenhagen ,1956, p.319

(8) ib.p.319

(9) ib. p.319

(10) ib.p.320

(11) For some interpretations in an Aristotelian and Platonic sense the following texts can be seen: P.Delsedine, *L'infini numerique dans l'Arenarie d'Archemède*, Ahes, 1970,*345-59, G. Gambiano, Scoperta e dimostrazione in Archimede , in Archimedes. Myth Tradition Science*, Olski, Florence 1992, pp.21-41, Virieux Reymond, *Le platonisme d'Archimedes*, Revue philosophique , 169,1979, pp.189-92. Not to speak about Frajese of whose text Archimedes' presumed Platonism pours in all parts

(12) See T. L. Heath, *The works of Archimedes, with a Supplement The method of Archimedes*, Cambridge, 1912.

(13) See ib. p.314

(14) See C. Mugler, *Les oeuvres d'Archimède*, Paris,1970, pp.83-84

(15) See P. Ver Eecke*, Les oeuvres complètes d'Archimède*, Paris, 1959.

(16)See A. Frajese, *Opere di Archimede* , Utet, Torino, 1974,same as above pp.572-573 and others

(17)See *Appendix*,

(18) See Heath p.12

(19) See Dijksterhuis p. 320 op.

(20)See Heath pp17-18 …

(21)See T.L. Heath. *The works* …, p. 152-53

(22)See Knorr's essay in addition to the cited work *Archimede. Mito Tradizione Scienza*, p. 353

(23) For a harsh judgement about Plato, witness of mathematical things, I recall a judgement of Peano, who certainly knew about mathematics, as with Greek philology: "*Untrue in this philosopher's dialogues there are some mathematical terms here and there, but reunited in such an uncertain way that they were considered as difficult words which an interlocutor looks for to confuse his adversary; almost like in our political newspapers of today **incommensurable** is written instead of **very big**"* . (See Giuseppe Peano, *Opere*, Vol. III,Roma,1958, p.249).

(24) Laertius D. trans. R. D. Hicks , *Lives of Eminent Philosophers, I*, Trans. R. D. Hicks, 1925. Harvard University Press 1925

(25) See Proclus, *Comment on the first book of Euclid's elements,* prologue, cap.iv

(26)As Frajese supports, taken from G.Giorello in *Archimede e la metodologia dei programmi di ricerca* , Scientia, 1975,p.118

(27)See A. Frajese, *Opere di Archimede*,p.70

(28)See the preSocratics ,44,b,4

(29) It is well put into evidence that the Aristotelian logic is far from mathe-
matical logic from the following passages by the logician and mathematician
Peano, which moreover criticizes even the scholastic logic in these passages
while regarding the Aristotelian logic, that one being the continuation of this
one:

"*It is known that scholastic logic is not of appreciable use in mathematical
demonstrations ,since in these there hare never mentioned the classifications
and the rules of syllogism*" . "**Mathematical logic** *is the science that treats
forms of reasoning which are met in the various mathematical theories reduc-
ing them to forms similar to algebraic ones. The only thing that it has in
common with the Aristotelian logic is syllogism. The classifications of the dif-
ferent ways of syllogisms , when they are precise, are of little importance in
mathematics. In mathematical sciences one comes across numerous forms of
reasoning not reducible to syllogisms".* See Giuseppe Peano, *Opere scelte*,
vol.II Roma, Cremonese, p.80 and p.379.

(30) *"Democritus calls the atoms the full"* (pleres). See S.Luria, Democri-
tus,197,

(31)See S.Luria, Democritus, 212 : *The term indivisible is used in many
meanings. It could mean 1) that it hasn't yet been divided, but could be di-
vided just like each of the continuous magnitudes, or else, 2) it is totally indi-
visible due to its own nature lacking parts enabling its division, like the point
or the one, or more so, 3) that it possesses **both parts and a magnitudes, but
it is not passable because of its solidity and density as in the case of
Democritus' atoms***": Simplicius,*Physics*,I,2,b 8

(32) See *The preSocratics* , 68,b,47

(33) For a history of the two traditions of thought see Giuseppe Boscarino,
*Tradizioni di pensiero. La tradizione di pensiero italica della scienza e della
realtà* , Sortino, 1999. For the influence of the italic school on Archimedes'
physics, see S. Notarrigo, *Archimede e la fisica* in *Archimede. Mito
Tradizione Scienza,* cit. ,pp.381-394.

(34) See T.L. Heath. *The works* …, p.151

(35) See Enrico Giusti, *Ipotesi sulla natura degli oggetti matematici*, Borin-
ghieri, Torino,1999, p.76

(36) See Giuseppe Boscarino, *Le forme e i mutamenti della scienza. Oggetti-
vità scientifica e tradizioni di pensiero.* Mondotre - la Scuola italica , 2004,
n.5

(37) For a more formal and less philosophical reading on the problem of the indivisibles please see *Ripensano Peano e la sua scuola*, by Giuseppe Boscarino, still Giuseppe Boscarino's part, *Gli indivisibili di Cavalieri alla luce dell'insegnamento di Peano*, in Mondotre, 1989,pp.27-33

Appendix

Archimedes' method (*Archimedous éphodos)* **on mechanical theorems** (*perì ton mekhanikon theoremathon*) **to Eratosthenes** (*pros Eratosthenen*)

18-28, 1-9

But seeing that you *(Oròn dè se),* as much as I state (*kathaper légo*), are zealous and in an excellent way master of philosophy (*spoudaion kai philosophias aksiologos*)and that you also know whow to evaluate *(kai tetimekota)* in mathematical things*(en tois mathemasin)* <observation>(**then theorian**) that is presented to you (*katà tò upopipton*), decided (*edokimasa*) to write to you (*grapsai to*) and to in the same book expose (*eksorisai is tò autò* bibion) the properties of **a certain way₁ (** *idioteta **tropou tinos***) through which (*kat'on*)will be given to you (*estai parekhomenon soi*) to grasp the means(*lambanein aphormas)* in order to have the ability (*eis tò dunasthai*) **to <observe>** (**theorein**) some of them(*tina ton)* in mathematical things(*en tois mathemasi)* by means of mechanical beings (**dià ton mekhanikon**).

I am then convinced (*pepeismai dè*) that this (touto)is not less (*einai ouden esson),* even in the demonstration(*kai eis ten apodeiksin)* of the same theorems (*auton ton theorematon*).

In fact, some of the things(*tina ton*) that have **appeared** to me(*moi phanenton***)** firstly in a **mechanical way₂** (*proteron* **mekhanikòs**), then (*usteron*) were exposed in a **geometrical way₃ (geometricòs)**, because **the observation** (*dià tò ten theorian*) through **this way₄** (**dià toutou tou tropou**) is **without a demonstration** (*korìs opodeikseos*, geometrical; ours added); is by fact easier (*etoimoteron gar esti)* having first obtained(*prolabonta)* by means of **this way₅**(**dià tou tropou**) a somewhat sort of knowledge (**gnosin tina)** of things searched (*zetemoton*) for provide the demonstration (*porisastai ten apodeiksin*) rather than (*mallon*) looking for(*zetein*), having no preliminary knowledge(*e medenos egnosménou*).

Therefore, of those theorems (*Dioter kai ton theorematon touton*), of the which(*on*) Eudoxus (*Eudoksos*)found as the first one the demonstration

(*ekseureken protos ten apodeiksin*)) on the cone and the pyramid(*perì tou konou kai tes puramidos*) that (*oti*) the cone is the third part of the cylinder (*o men konos triton meros tou kulindrou*), the pyramid of the prism(*e de puramis tou prismatos*), having (*ton ekhonton*) the same base and equal height(*ten auten basin kai upso ison s*), not a small part could be attributed to Democritus(*ou micron merida aponeimai an tis Democrito*) as the first person to introduce the knowledge of assertion(*proto apophenameno ten apophasin ten* < regarding the knowledge = *mathesin*.; ours added> on the above mentioned figure(*perì tou eiremenou skhematos)* without any <geometrical, ours>demonstration (*khoris apodeikseos*).

Thus, we before write(*gràfomen oun proton*)the first of the theorems(*to kai proton*), appeared (**phanèn**) by means of mechanical beings (*dià ton mekhanikon*) that any segment of a section of a right –angled cone(*oti pan tmema orthogonìon konou tomès*) is four herds of the triangle(*epitripton estin trigonou tou*) which has(*ekhentos)* the same base and equal height (*ten auten basin kai upsos ison*), after (**metà**) each one(*eskaton ton*) being **observed** *(theorethenton)* in the **same way₆(dià tou autou tropou)** ; and so at the end of the book (*epì telei de tou bibliou)* we write the geometrical demonstrations(*gràfomen tas geometrikas apodeikseis*) of those theorems(*ekeinon ton theorematon*), of which we had previously sent the enunciated (*on apeisteilamen soi proteron protaseis*).