

Renè Thom: forms, catastrophes and complexity

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ABSTRACT. According to René Thom, his catastrophe theory, a great interpretative and explicative framework of the structure of reality in mathematical-topological qualitative terms, differs from usual mathematical theories. In fact, by opposing the purely quantitative view of mathematics, both Thom and Aristotle used the concept of boundary to define reality through the forms it assumes, so also allowing to build analogies between different things and concepts. Starting from here, Thom arrived at the so called “cobordism”, consisting in the continuous crossing between different spaces, even of different dimensions, and then to few “catastrophes” or abrupt, although always continuous, universal forms. The study of forms in irregular, accidental and even chaotic situations had already started at the end of the XIX Century, to single out catastrophic evolutions as structure invariants in terms of divergences due to sensitive dependence on little variations of the initial conditions. The theory of catastrophes then provides a model or qualitative mechanism rather than equations describing and predicting changes quantitatively, as it is not expected that to deal mathematically only equals to quantify rather than to put in relation. Anyway, even though expressed mathematically, an analogy is a qualitative rather than a quantitative relation, as Aristotle had already affirmed before Thom. The philosophy which expresses itself here is in any case for Thom more difficult than the mathematics which in case it uses, as it is a metaphysics carrying out a demand of universal unification of reality.

1. From cobordism to catastrophe theory

As is well known to the readers of his essay *Prédire n'est pas expliquer*, René Thom thought that the academic job of refining/applying mathematical theories was quite different from his catastrophe theory, meant as a great interpretative/explicative framework of the structure of reality in topological/qualitative terms. With it in fact Thom intended to overcome the purely quantitative conception of mathematics (meant as mere computation and exact forecasting instrument) into its different conception as a universal explicative code, reconciled with metaphysics, especially with Aristotle's ontological metaphysics, wrongly considered contrary to the mathematical treatment of reality because of its refusal to reduce knowledge to mere quantitative measurement. In particular, like Aristotle, Thom utilizes the border/edge concepts in order to define both physical and mental individual realities through the forms delimiting them (which may be just represented in geometrical-topological terms).

Reality would then show itself through the borders delimiting it, the distinguished forms it assumes, so giving rise to geometrical notions linked to concrete experience. These forms also allow one to build formal analogies between things and concepts, by sending, says Thom, a space into another. Starting from here Thom then studied the continuous crossing from a variety (space) to another, the connections through common boundaries and points between spaces even endowed with different dimensions (a research on the so called "cobordism" which yielded him the Field Medal in 1958), until he singled out few universal forms, that is mathematical objects representing catastrophes or abrupt, although continuous, transitions of forms: specific singularities appearing when an object is submitted to bonds, such as restrictions with regard to its ordinary dimensions, that it accepts except in particular points where it offers resistance by concentrating there, so to say, its structure. Examples thereof are corners in solid objects where, so to say, catastrophes are crystallized in static form and then memorized as a part of their history, while they vanish as soon as they are formed in the fluids, which are endowed with scant memory.

2. Catastrophes, continuity and complexity

Seven are the elementary types of catastrophes or generic singularities of an application and Thom decided to study their applications in caustics, surfaces

lit according to different angle shots, reflections and refractions. Initially catastrophe theory was of use just to explain caustic formation and only afterwards many other phenomena, but without yielding quantitative solutions and exact predictions, rather qualitatively framing situations that were uncontrollable by only reductionistic quantitative methods summing up elementary units. The study of forms in irregular, accidental and even chaotic situations had truly in advance led scientists like J.H.Poincaré and J. Hadamard, between the XIX and XX centuries, to single out structurally invariable catastrophic evolutions in the most disparate phenomena, in terms of divergences due to sensitive dependence on little variations of the initial conditions. In such cases there were not exact laws, rather evolutionary asymptotic tendencies, which did not allow exact predictions, in case only statistic ones. While when exact predictions are possible, in terms of strict laws and explicit equations, the catastrophe model is no more of use.

To be true, it is possible to proceed, according to circumstances, either actively trying to make exact predictions, or contemplatively describing catastrophic global processes. The mathematical function concept, typical of modern science at least starting from its first accurate formulation by Leibniz in 1695 is, in particular, identifiable with reality universal determinism. I. Prigogine's pretension to deny determinism is for Thom unfounded as it is not possible to do without it, even when we face, though unwillingly, seemingly undetermined situations, which will prove themselves determined. As to complexity, in particular in biology contrary to reductionism, a finalized global macroscopic organism is easier to understand than a single cell, which is in fact blind and casual. Then it is better to proceed on the contrary, by maintaining determinism rather than reductionism, from the global observation of organisms great structures to their decomposition into parts and to the finer and finer description of local structures. Anyway, catastrophe theory studies forms as qualitative discontinuities though on a continuous substrate. In any case forms as mental facts are immersed in a matter which is still a thought object. The more you try to analyze it the more it appears as a fog, revealing a more and more complex and inexhaustible weaving the more it refines itself through the forms it assumes. In fact complexity is more and more ascertained until a true enigma is reached when you once for all want to define reality as a universe endowed with a high number of dimensions and then object of mental experiences to which even objective phenomena are at the end concretely reduced. Concrete reality is yet more evident than a scientific explanation and naïve ontology appears more concrete than the scientific one. It is steady and

universal, while the latter is always problematic and revisable. Besides, according to G. Bachelard, while naïve explanation is immediately reflected into the ordinary language which is accessible to everybody, the claimed scientific explanation goes with its jargon beyond immediate experience, away from the life world which only we can know immediately.

As for example, the continuous character of reality, which Thom entrusts to a world intuition as a frame of the phenomenological discontinuities themselves, is instead contradicted by the present tendency to reduce all to discrete units of information (bits) of modern computing. Of course it has a practical value: an animal individuating a prey perceives it as an entity which is absolutely distinct from its environment, just as we discretize linguistic phonemata to learn speaking without confounding them. Yet a continuous background remains, notwithstanding the tendency of our brains to discretize. Such background is for example constituted by space and time. Continuum is said an illusion as exemplified by a film which appears continuous to us, while it is made of discrete frames. Really it is an illusion but with a true mental base, otherwise it would not arise at all, and such base is just the existence of continuum. Really we perceive continuum but need discreteness, finiteness in order to keep things under control. Anyway quantum mechanics seems to introduce discreteness in absolute terms, something we do not understand but which is operatively valid, as is shown by the possibility to localize or delocalize a wave packet by simply varying the value distributions of complementary variables as position and momentum or time and energy, according to Heisenberg's principle of indetermination. Anyway, also the apparent quantum discontinuity hides a continuity which, always according to Heisenberg's principle, may be only obscured and not cancelled in several phenomena. It is difficult to conceive but not monstrous. The hypothesis according to which we are finite and discrete in our internal structure is afterwards false for we are more than that. We have hundreds billions of neurons, which are in continuous movement, as they are constituted by molecules continuously vibrating in the space, so giving place to infinite possible variations in a considerable dimensions number, even though we are reduced to the smallest possible number of states and dimensions to deal with the system under study, according to a technical and algorithmic thought which is operatively effective, certainly practically motivated but unidentifiable with reality.

Science looks for solutions, but often finds itself in front of difficulties which reveal unsolved the problems, illusory the solutions. In mathematics and physics the difficulty is formalized and then dimmed, while in biology it

appears dramatic as a metaphysical problem, as for example when it is affirmed that life is reducible to the physical and chemical laws of unanimated matter, against immediate intuition rather suggesting an irreducibility. But also in mathematics impressive difficulties are found, as Goedel's theorem according to which if you want to demonstrate the non contradiction of arithmetic in the admitted conceptual frame, you arrive at demonstrating that this non contradiction is not demonstrable. At most you can clarify local aspects referring to the foundations of mathematics, rather than the global problem of foundations, always returning to the discreteness-continuum opposition. Continuum is in fact the universal substrate of all thought, especially of the mathematical one, but nothing which is effective can be thought without assuming discreteness in this continuity.

3. Quantity and quality: science, mathematics and philosophy

Catastrophe theory then yields a model or qualitative mechanism rather than equations describing and quantitatively predicting variable changes, deformations, perturbations, while catastrophe theory expresses invariant forms of such perturbations. From there many critiques have arisen, starting from Rutherford's sentence: "Qualitative is a scanty quantitative". Others have said: "Catastrophe theory produces only metaphors". In fact it leaves aside the physical, biological or chemical nature of phenomena and does not consider them like physical ones in general, as ruled by exact quantitative laws, it cannot mathematize in current physical terms what is not mathematizable in quantitative terms. As is well known also determinists like Leibniz do not take for granted that to mathematize means to quantify any phenomena rather than to put relations in general. For Thom, in fact, few natural phenomena are ruled by exact quantitative laws, and they are physical, while almost all the other laws of phenomena, even though quantitative, are only approximate. Catastrophe theory then yields intelligibility outlines, which might be also deceptive. By the way, according to Konrad Lorenz any analogy is true, but maybe it is more correct to say that any analogy is true provided that it is semantically acceptable, that is if to a purely mental analysis it results correct, provided the mind recognizes that it is truly correct. Anyway it is a strictly qualitative relation and can be expressed mathematically, even though its evaluation does not necessarily depend on its mathematical form. As for example, to say with Aristotle that the evening is the old age of the day and the

old age is the evening of the life, means to mentally, semantically elaborate an analogy in two formulations, the latter of which moreover asserts itself as more convincing for the mind than the former, notwithstanding the simple proportionality, the equality of two ratios which the analogy expresses from the purely mathematical point of view. Anyway its structure implies the fundamental notion of edge or end. In fact it is about a time interval, of which the edge or end is enunciated as evening or old age, whose corresponding catastrophe is a fold: a stable regime (life or day) meets an instable one (old age or evening).

Anyway, the philosophy which expresses itself here is for Thom more difficult than mathematics: it asks for more theoretical efforts, as it is less precise and delimited, surely more complex and requiring a much harder training. Besides, according to Thom mathematics is instead by now a remunerative job at least since Napoleon, then its motivations are no more linked to a disinterested pleasure stimulated, for example, by knowledge popularization, which is then no more a common tool of recruitment of researchers. As to scientific explanation, at the end it is reduced to describe a phenomenon, as for example the collision of two plates as a cause of an earthquake, which does not concern the victims of the catastrophe. It is then necessary to go back to the reasons of the collision until the first cause which is not explained at its turn. For Thom once again Aristotle's metaphysics gives a smart solution to the problem, by conceiving God as first uncaused cause. Anyway, at the base of the physical-mathematical synthesis, there is for Thom the idea of a creator/organizer realizing a universal unification need. For Nietzsche, notes Thom at the end, new ideas arrive always on the legs of a pigeon, so unifying parts which are also very far from one another through their displacements, in an almost imperceptible way without almost arousing echoes.

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