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A Note on Confirmation and Matthew Properties

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ABSTRACT. There are numerous (Bayesian) confirmation measures in the literature. Festa provides a formal characterization of a certain class of such measures. He calls the members of this class "incremental measures". Festa then introduces six rather interesting properties called "Matthew properties" and puts forward two theses, hereafter "T1" and "T2", concerning which of the various extant incremental measures have which of the various Matthew properties. Festa's discussion is potentially helpful with the problem of measure sensitivity. I argue, that, while Festa's discussion is illuminating on the whole and worthy of careful study, T1 and T2 are strictly speaking incorrect (though on the right track) and should be rejected in favor of two similar but distinct theses.

KEYWORDS: Confirmation, Festa, Matthew Properties, Problem of Measure Sensitivity.

1. Introduction

There are numerous (Bayesian) confirmation measures in the literature. Festa (2012) provides a formal characterization of a certain class of such measures.[1]

[1] All references to Festa are to Festa (2012).

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He calls the members of that class "incremental measures".[2] Each of the following is an incremental measure:[3]

$$c_d(H, E) = p(H|E) - p(H)$$

$$c_{lr}(H, E) = \frac{p(E|H)}{p(E|\neg H)}$$

$$c_r(H, E) = \frac{p(H|E)}{p(H)}.$$

Festa then introduces six rather interesting properties called "Matthew properties" and puts forward two theses, hereafter "T1" and "T2", concerning which of the various extant incremental measures have which of the various Matthew properties.

No two of the three measures c_d, c_{lr}, and c_r are ordinally equivalent to each other (i.e., impose the same ordering on any two ordered pairs of proposi-

[2] Below, in Section 2, I explain Festa's formal characterization of the class of incremental measures. It is worth noting now, though, that *incremental* measures are more than just *relevance* measures, where a measure c is a relevance measure just in case there is a neutral point n such that $c(H,E) > / = / < n$ iff $p(H|E) > / = / < p(H)$. (This characterization of relevance measures is adapted from Fitelson 1999.) Consider the following (well known) measures:

$$c_C(H,E) = p(H \wedge E) - p(H)p(E)$$

$$c_M(H,E) = p(E|H) - p(E)$$

$$c_N(H,E) = p(E|H) - p(E|\neg H)$$

$$c_S(H,E) = p(H|E) - p(H|\neg E).$$

Each of these measures is a relevance measure as defined above. But none of them is an incremental measure as characterized by Festa. This is just as it should be, it seems, if an incremental measure is understood as a measure of the amount of increase in H's probability due to E, for on each of c_C, c_M, c_N, and c_s there can be cases where $p(H_1|E_1) > p(H_2|E_2)$ while $p(H_1) < p(H_2)$ and yet the degree to which E_1 confirms H_1 is less than the degree to which E_2 confirms H_2. This allows that there are conceptions of confirmation distinct from the incremental conception (where confirmation is a matter of the amount of increase in H's probability due to E) and in terms of which c_C, c_M, c_N, and c_s are best understood. See Hajek and Joyce (2008) and Joyce (1999, Ch. 6, sec. 6.4) for relevant discussion.

[3] The subscripts in these measures, along with the subscripts in the measures set out below in Section 2, are identical to the subscripts used by Festa.

tions).[4] This is prima facie problematic in that each of the three measures has some intuitive plausibility and yet certain results in confirmation theory involving one of the measures do not carry over to (at least one of) the other two measures. This is the problem of measure sensitivity.[5]

Festa's discussion is potentially helpful with this problem. Suppose one of the various Matthew properties is compelling in that any adequate incremental measure should have that property. Suppose it follows from T1 and T2 that, say, c_d has the property in question but neither c_{lr} nor c_r does. Then c_{lr} and c_r should be rejected as inadequate (as incremental measures). This would serve to narrow down the field of potentially adequate incremental measures and thus constitute progress towards solving the problem of measure sensitivity.

It turns out, however, that, while Festa's discussion is illuminating on the whole and worthy of careful study, T1 and T2 are strictly speaking incorrect (though on the right track). In Section 2, I set out the various incremental measures under consideration along with T1 and T2. In Section 3, I argue that T1 and T2 should be rejected in favor of two similar but distinct theses. In Section 4, I conclude.

2. Festa's Two Theses

Festa characterizes the class of incremental measures in terms of the following properties (or conditions):[6]

Initial and Final Probability Dependence (IFPD): $c(H,E)$ is a function of $p(H|E)$ and $p(H)$.

Final Probability Incrementality (FPI): Suppose $p(H_1) = p(H_2)$. Then $c(H_1,E_1) > / < c(H_2,E_2)$ if and only if $p(H_1|E_1) > / < p(H_2|E_2)$.

[4] Measures c and c^* are ordinally equivalent to each other just in case, for any ordered pairs of propositions $<H, E>$ and $<H', E'>$, the following holds: $c(H, E) > / = / < c(H', E')$ iff $c^*(H, E) > / = / < c^*(H', E')$.

[5] See Brössel (2013) and Fitelson (1999) for helpful discussion of the problem of measure sensitivity.

[6] It should be understood throughout the discussion that the propositions involved in the various probabilities are "p-normal" in that they have nonextreme unconditional probabilities (i.e., unconditional probabilities less than one and greater than zero).

Initial Probability Incrementality (IPI): Suppose $0 < p(H_1|E_1) = p(H_2|E_2) < 1$. Then $c(H_1,E_1) > / < c(H_2,E_2)$ if and only if $p(H_1) < / > p(H_2)$. Suppose $p(H_1|E_1) = p(H_2E_2) = 0$ or $p(H_1|E_1) = p(H_2|E_2) = 1$. Then (a) $c(H_1,E_1) \geq c(H_2,E_2)$ if $p(H_1) < p(H_2)$ and (b) $c(H_1,E_1) \leq c(H_2,E_2)$ if $p(H_1) > p(H_2)$.

Equineutrality (E): Suppose $p(H_1|E_1) = p(H_1)$ and $p(H_2|E_2) = p(H_2)$. Then $c(H_1,E_1) = c(H_2,E_2)$.

The class of incremental measures is defined as the class of measures having each of IFPD, FPI, IPI, and E.

It turns out that many extant confirmation measures are members of the class of incremental measures. Festa considers, in addition to c_d, c_{lr}, and c_r, the following:

$$c_{r*}(H,E) = \frac{p(H|E) - p(H)}{p(H|E) + p(H)}$$

$$c_{or}(H,E) = \frac{o(H|E)}{o(H)} \text{ where } o(H|E) = \frac{p(H|E)}{p(\neg H|E)} \text{ and } o(H) = \frac{p(H)}{p(\neg H)}$$

$$c_G(H,E) = \frac{p(H|E) - p(H)}{1 - p(H)}$$

$$c_z = \begin{cases} \dfrac{p(H|E) - p(H)}{1 - p(H)} & \text{if } p(H|E) \geq p(H) \\[2ex] \dfrac{p(H|E) - p(H)}{p(H)} & \text{if } p(H|E) < p(H) \end{cases}$$

$$c_{So}(H,E) = \frac{\log[p(H|E)/p(H)]}{-\log[p(H)]}$$

$$c_{Pl}(H,E) = \frac{p(H|E) - p(H)}{p(H|E) + p(H) - p(H|E)p(H)}$$

$$c_{hP}(H,E) = \frac{p(H|E) - p(H)}{p(H|E) + p(H) + p(H|E)p(H)}$$

$$c_{\pi}(H,E) = \frac{p(H|E) - p(H)}{p(H|E) + p(H) + \pi p(H|E)p(H)} \text{ where } -2 \leq \pi \leq \infty$$

$$c_{\alpha}(H,E) = \frac{p(H|E) + \alpha p(H)p(H|E)}{p(H) + \alpha p(H)p(H|E)} \text{ where } -1 \leq \alpha \leq \infty$$

$$c_{db}(H,E) = \frac{p(H|E) - p(H)}{p(H|E)p(H)}$$

$$c_{KO}(H,E) = \frac{p(E|H) - p(E|\neg H)}{p(E|H) + p(E|\neg H)}$$

$$c_{P}(H,E) = \frac{p(E|H) - p(E)}{p(E|H) + p(E) - p(H \wedge E)}$$

$$c_{Ku}(H,E) = \frac{p(E|H)}{p(E)}$$

Some of the sixteen measures under consideration are ordinally equivalent to each other: c_r is ordinally equivalent to each of c_{Ku} and c_{r*}; c_{lr} is ordinally equivalent to each of c_{or} and c_{KO}; c_P is ordinally equivalent to c_{Pl}.[7] This is significant in that if one measure is ordinally equivalent to another, then the one has a given Matthew property just in case the other too has that property. I thus want to set aside c_{Ku}, c_{r*}, c_{or}, c_{KO}, and c_{Pl} and focus on c_r, c_{lr}, and c_P along with the remaining eight measures.[8]

Take some incremental measure c. Since c has IFPD, it follows that $c(H, E)$ is a function of $p(H|E)$ and $p(H)$. But:

$$p(H,E) = \frac{p(E|H)}{p(E)} p(H).$$

So $c(H,E)$ is a function of $Q(H,E) = p(E|H)/p(E)$ and $p(H)$, where, following Festa, $Q(H,E)$ is H's predictive success with respect to E.

[7] It is straightforward to verify that $c_r(H,E) = c_{Ku}(H,E)$, $c_{r*}(H,E) = [c_r(H,E) - 1] / [c_r(H,E) + 1]$ where $[n - 1] / [n + 1]$ is an increasing function of n for $n \geq 0$, $c_{lr}(H,E) = c_{or}(H,E)$, $c_{KO}(H,E) = [c_{lr}(H,E) - 1] / [c_{lr}(H,E) + 1]$ where, again, $[n - 1] / [n + 1]$ is an increasing function of n for $n \geq 0$, and $c_P(H,E) = c_{Pl}(H,E)$.

[8] There is thus no mention of c_{Ku}, c_{r*}, c_{or}, c_{KO}, and c_{Pl} in T1 and T2 as formulated below.

It follows, as Festa notes, that each of the incremental measures under consideration can be restated in terms of $Q(H,E)$, hereafter "Q", and $p(H)$. $c_d(H,E)$, for example, can be restated as $p(H)[Q - 1]$. This can be seen as follows:

$$\begin{aligned} c_d(H,E) &= p(H|E) - p(H) \\ &= \frac{p(E|H)}{p(E)}\, p(H) - p(H) \\ &= p(H)[Q - 1] \end{aligned}$$

Festa provides a "Q-function" for each of the incremental measures under consideration.

I can now state the six Matthew properties introduced by Festa. They can be put as follows:

Matthew Independence for Positive Confirmation (MIP): For any $Q > 1$, if Q is held fixed, then $c(H,E)$ is held fixed and thus is independent of $p(H)$.

Matthew Effect for Positive Confirmation (MEP): For any $Q > 1$, if Q is held fixed, then $c(H,E)$ is an increasing function of $p(H)$.

Reverse Matthew Effect for Positive Confirmation (RMP): For any $Q > 1$, if Q is held fixed, then $c(H,E)$ is a decreasing function of $p(H)$.

Matthew Independence for Disconfirmation (MID): For any $Q < 1$, if Q is held fixed, then $c(H,E)$ is held fixed and thus is independent of $p(H)$.

Matthew Effect for Disconfirmation (MED): For any $Q < 1$, if Q is held fixed, then $c(H,E)$ is a decreasing function of $p(H)$.

Reverse Matthew Effect for Disconfirmation (RMD): For any $Q < 1$, if Q is held fixed, then $c(H,E)$ is an increasing function of $p(H)$.

Recall that (following Festa) Q is H's predictive success with respect to E. MIP can be glossed: for any degree of predictive success greater than 1, $c(H,E)$ is independent of H's prior probability. MEP, in turn, can be glossed: for any degree of predictive success greater than 1, the greater is H's prior probability, the greater is $c(H,E)$. And so on for RMP, MID, MED, and RMD.

Why are the six Matthew properties named "Matthew" properties? Festa (referencing Kuipers 2000) writes:

> Kuipers ... introduces the concept of *Matthew effect for confirmation* just w.r.t. [MEP restricted to cases where H logically implies E]. In fact, [MEP restricted to cases where H logically implies E] "may be seen as a methodological version of the so-called Matthew effect, according to which the rich profit more than the poor" ..., in agreement with the sentence—made famous by the Gospel according to St. Matthew—that "unto every one that hath shall be given". (p. 95, emphasis original)

MEP implies that if two hypotheses have the same predictive success (greater than 1) with respect to some piece of evidence, and if initially the two hypotheses had different probabilities, then the hypothesis that initially had the higher probability (the "richer" of the two hypotheses initially) is more strongly confirmed by ("profits" more from) the evidence than does the hypothesis that initially had the lower probability (the "poorer" of the two hypotheses initially). Thus the name "Matthew Effect for Positive Confirmation" and, for consistency, the names of the remaining five properties.

It is clear that MIP, MEP, and RMP are pairwise mutually inconsistent in that any measure having one of them lacks each of the other two. It is also clear that the same is true with respect to MID, MED, and RMD. But which measures have which properties?

T1 and T2 are meant to answer this question. They can be put like this:

T1 a c_r has MIP and MID.
b c_d, c_G, c_{So}, c_{lr}, and c_P have MEP and MED.
c c_z has MEP and MID.

T2 a c_{hP} and c_{db} have RMP and RMD.
b c_π has MEP and MED when $\pi < 0$, has MIP and MID when $\pi = 0$, and has RMP and RMD when $\pi > 0$.
c c_α has MEP and MED when $\alpha < 0$, has MIP and MID when $\alpha = 0$, and has RMP and RMD when $\alpha > 0$.

T1 and T2, I take it, are meant to follow straightforwardly from the various Q-functions provided by Festa. Recall that the Q-function for $c_d(H, E)$ is $p(H)[Q - 1]$. If $Q > 1$ and Q is held fixed, it follows that $p(H)[Q - 1]$ is an increasing function of $p(H)$. If $Q < 1$ and Q is held fixed, it follows that $p(H)[Q - 1]$ is a decreasing function of $p(H)$. So, just as T1b implies, c_d has MEP and MED.

It turns out, however, that not all is right with T1 and T2. Some modifications are needed.

3. Two Replacement Theses

Suppose E entails $\neg H$ so that $p(H|E) = 0 = p(E|H)$. Then $p(H|E)/p(H) = 0$ regardless of $p(H)$. But $Q = p(E|H)/p(E) = p(H|E)/p(H)$. So $Q = 0$ regardless of $p(H)$. Suppose c is an incremental measure such that $c(H,E)$ takes the minimum value (for c) in any case where E entails $\neg H$. Then it is not true that for any $Q < 1$, if Q is held fixed, then $c(H,E)$ is a decreasing function of $p(H)$, and it is not true that for any $Q < 1$, if Q is held fixed, then $c(H,E)$ is an increasing function of $p(H)$. So c has neither MED nor RMD.

This spells trouble for T1 and T2. Suppose E entails $\neg H$. Then it follows that:[9]

$$
\begin{aligned}
c_{So}(H,E) &= \frac{\log[p(H|E)/p(H)]}{-\log[p(H)]} \\
&= \frac{-\infty}{-\log[p(H)]} \\
&= -\infty
\end{aligned}
$$

$$
\begin{aligned}
c_{lr}(H,E) &= \frac{p(E|H)}{p(E|\neg H)} \\
&= \frac{0}{p(E|\neg H)} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
c_{P}(H,E) &= \frac{p(E|H) - p(E)}{p(E|H) + p(E) - p(H \wedge E)} \\
&= \frac{0 - p(E)}{0 + p(E) - 0} \\
&= -1
\end{aligned}
$$

[9] c_{So} can be understood as having the range $(-\infty, 1]$. See Shogenji (2012, p. 37) and Atkinson (2012, p. 53). But then, as the only plausible candidate value for $c_{So}(H,E)$ to take when $p(H|E) = 0$ is $-\infty$, it follows that $c_{So}(H,E)$ is undefined when $p(H|E) = 0$. This is less than ideal, it seems, since there should be a degree of confirmation even when $p(H|E) = 0$. It seems preferable to understand

$$c_{hP}(H,E) = \frac{p(H|E) - p(H)}{p(H|E) + p(H) + p(H|E)p(H)}$$
$$= \frac{0 - p(H)}{0 + p(H) + 0}$$
$$= -1$$

$$c_{\pi}(H,E) = \frac{p(H|E) - p(H)}{p(H|E) + p(H) + \pi p(H|E)p(H)}$$
$$= \frac{0 - p(H)}{0 + p(H) + 0}$$
$$= -1$$

$$c_{\alpha}(H,E) = \frac{p(H|E) + \alpha p(H)p(H|E)}{p(H) + \alpha p(H)p(H|E)}$$
$$= \frac{0 + 0}{p(H) + 0}$$
$$= 0$$

Note that $c_{\pi}(H,E) = -1$ and $c_{\alpha}(H,E) = 0$ regardless of the values specified for π and α respectively. It follows that c_{So}, c_{lr}, and c_P do not have MED, that c_{hP} does not have RMD, that c_{π} does not have MED when $\pi < 0$ and does not have RMD when $\pi > 0$, and that c_{α} does not have MED when $\alpha < 0$ and does not have RMD when $\alpha > 0$. So T1b is incorrect and each of T2a, T2b, and T2c is incorrect. So T1 and T2 are incorrect.

T1 and T2, though, are on the right track. They can be replaced by the following:

T1* a c_r has MIP and MID.
b c_d and c_G have MEP and MED.
c c_{So}, c_{lr}, and c_P have MEP but do not have MID, MED, or RMD; c_{So}, c_{lr}, and c_P have MED in the special case where $1 > Q > 0$.
d c_z has MEP and MID.

c_{So} as having the range $[-\infty, 1]$ and as taking the value $-\infty$ when $p(H|E) = 0$. Atkinson and Shogenji (personal communication) agree that c_{So} should be understood as having the range $[-\infty, 1]$.

T2* a c_{db} has RMP and RMD.

b c_{hP} has RMP but does not have MID, MED, or RMD; c_{hP} has RMD in the special case where $1 > Q > 0$.

c c_{π} has MEP but does not have MID, MED, or RMD when $\pi < 0$; c_{π} has MIP and MID when $\pi = 0$; c_{π} has RMP but does not have MID, MED, or RMD when $\pi > 0$; c_{π} has MED when $\pi < 0$ in the special case where $1 > Q > 0$; c_{π} has RMD when $\pi > 0$ in the special case where $1 > Q > 0$.

d c_{α} has MEP but does not have MID, MED, or RMD when $\alpha < 0$; c_{α} has MIP and MID when $\alpha = 0$; c_{α} has RMP but does not have MID, MED, or RMD when $\alpha > 0$; c_{α} has MED when $\alpha < 0$ in the special case where $1 > Q > 0$; c_{α} has RMD when $\alpha > 0$ in the special case where $1 > Q > 0$.

T1* and T2* differ from T1 and T2 only with respect to cases where E entails $\neg H$ and thus $Q = 0$.

Some of the measures referred to in T1 and T2 have a maximum value and take that value in any case where E entails H. Consider c_{So} for example. If E entails H so that $p(H|E) = 1$, it follows that $c_{So}(H,E)$ takes its maximum value of 1 regardless of H's prior probability. Why is it that T1 and T2 run into trouble in the case where E entails $\neg H$ but do not run into trouble in the case where E entails H?

Return to the case where E entails $\neg H$. The key here is that Q equals 0 regardless of H's prior probability. This means that Q can be held fixed while $p(H)$ increases or decreases. This in turn means that if $c(H,E)$ takes the minimum value (for c) in any case where E entails $\neg H$, then there can be cases where E entails $\neg H$, Q is held fixed, and $c(H,E)$ remains constant at the minimum value while $p(H)$ decreases, in which case c does not have MED, and there can be cases where E entails $\neg H$, Q is held fixed, and $c(H,E)$ remains constant at the minimum value while $p(H)$ increases, in which case c does not have RMD. Things are different in the case where E entails H. Suppose c is an incremental measure such that $c(H,E)$ takes the maximum value (for c) in any case where E entails H. Suppose E entails H so that $p(H|E) = 1$. Then $1/p(H) = p(H|E)/p(H) = p(E|H)/p(E) = Q$. But then if Q is held fixed, it follows that $p(H)$ too is held fixed. Hence there can be no cases where E entails H, Q is held fixed, and $c(H, E)$ remains constant at the maximum value while $p(H)$ increases or decreases. So no case where E entails H could show that c does not have MEP or that c does not have RMP.

4. Conclusion

T1 and T2 are incorrect in some of what they imply with respect to cases where E entails $\neg H$ and thus $Q = 0$. They should be rejected in favor of T1* and T2*. The way is now clear for confirmation theorists to focus on which, if any, of the various Matthew properties are compelling.[10] By doing so confirmation theorists can perhaps use T1* and T2* to narrow down the field of potentially adequate incremental measures and so make progress towards solving the problem of measure sensitivity.

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[10] Festa (sec. 3.3.2) suggests that at least in some cases RMP is compelling.