Calculating noise figure in op amps

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Introduction

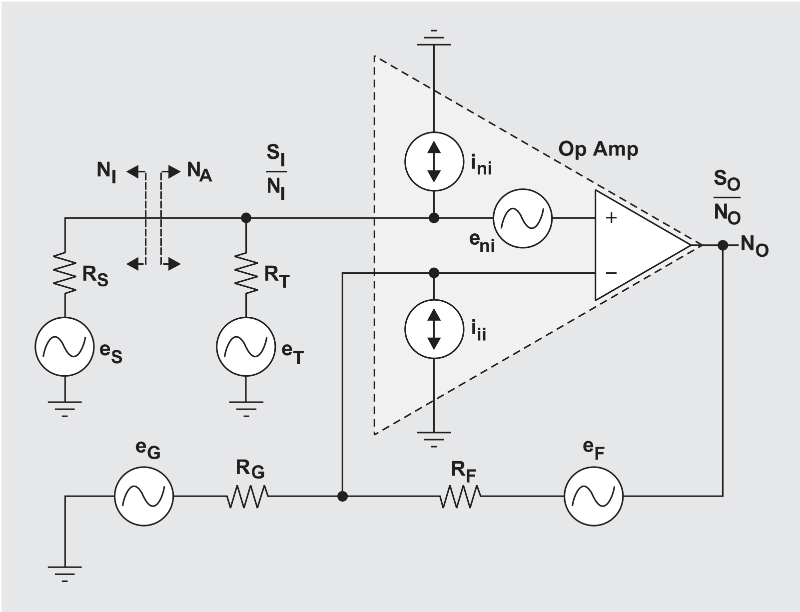
Noise figure is commonly used in communications systems because it provides a simple method to determine the impact of system noise on sensitivity.

Today, the performance of wide-band op amps is making them viable alternatives to more traditional open-loop amplifiers like monolithic microwave integrated circuits (MMICs) and discrete transistors in communications design.

Recognizing the need to specify wide-band op amps in RF engineering terminology, some manufacturers do provide noise figure, but they seem to be the exception rather than the rule.

Op amp manufacturers typically specify noise performance by giving the input-referred voltage and current noise. The noise figure depends on these parameters, the circuit topology, and the value of external components. If you have all this information, noise figure can be calculated.

Figure 1. Non-inverting noise analysis diagram



Review of noise figure

Noise figure (NF) is the decibel equivalent of noise factor (F): NF (dB) = 10log(F).

Noise factor of a device is the power ratio of the signal-to-noise ratio (SNR) at the input (SNR_I) divided by the SNR at the output (SNR_O):

$$F = \frac{SNR_I}{SNR_O}. \tag{1}$$

The output signal (S_O) is equal to the input signal (S_I) times the gain: $S_O = S_I \times G$. The output noise is equal to the noise delivered to the input (N_I) from the source plus the input noise of the device (N_A) times the gain: $N_O = (N_I + N_A) \times G$. Substituting into Equation 1 and simplifying, we get

$$F = \frac{SNR_I}{SNR_O} = \left[\frac{\frac{S_I}{N_I}}{\frac{G \times S_I}{G(N_I + N_A)}}\right] = 1 + \frac{N_A}{N_I}. \tag{2}$$

Assuming that the input is terminated in the same impedance as the source, $N_I = kT = –174$ dBm/Hz, where k is Boltzman's constant and T = 300 Kelvin). Once we find the input noise spectral density of the device, it is a simple matter to plug it into Equation 2 and calculate F.

NF in op amps

Op amps specify input-referred voltage and current noise. Using these two parameters, adding the noise of the external resistors, and calculating the total input-referred noise based on the circuit topology, we can calculate the input spectral density and use it in Equation 2.

In this discussion, the terms "op amp" and "amplifier" mean different things. "Op amp" refers to only the active device itself, whereas "amplifier" includes the op amp and associated passive resistors that make it work as a usable amplifier stage. In other words, the amplifier is everything shown in Figures 1–3 except R_S, and the op amp is only the components within the dashed triangles. In this way, the plane marked N_A and N_I is the input to the amplifier. This is the point to which the noise sources must be referred so that Equation 2 can be used.

The noise from the source and the input noise of the amplifier are referred to the same point. Because the impedance is the same, expressing the ratio between N_A and N_I as a voltage ratio squared is equivalent to the power ratio. An op amp is a voltage-driven device, so using voltage-squared terms makes the calculations easier. In the following discussion, voltage-squared terms are used for N_A and N_I.

Op amps use negative feedback to control the gain of the amplifier. One result is that the voltage across the input terminals is driven to zero. This is often referred to as a "virtual short." It is used in the following analysis* and referred to as "amplifier action," since it is a by-product of the op amp doing its job as an amplifier.

Superposition is used throughout the analysis, wherein all sources except the one under consideration are defeated—voltage sources are shorted and current sources are opened.

Non-inverting amplifier

Of the three basic op amp circuits, it is easiest to find the input-referred noise for the non-inverting op amp amplifier, so it will be discussed first. Figure 1 shows a noise analysis diagram for a non-inverting op amp amplifier with the noise sources identified.

The source resistance R_S generates a noise voltage equal to $\sqrt{4kTR_S}$. The noise voltage delivered to the amplifier input from the source is divided by the resistors R_S and R_T. Therefore,

$$N_I = 4kTR_S\left(\frac{R_T}{R_S + R_T}\right)^2.$$

R_T is typically used to terminate the input so that $R_T = R_S$, in which case $N_I = kTR_S$.

The amplifier's voltage noise is a combination of e_{ni}, i_{ni}, and i_{ii} with associated impedances e_T, e_G, and e_F. These are all referred

Figure 2. Inverting noise analysis diagram

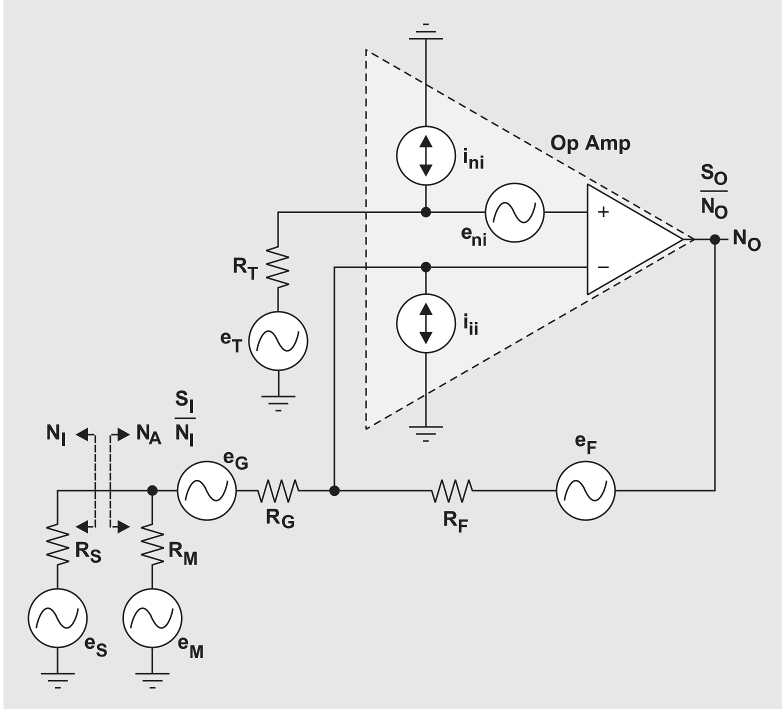
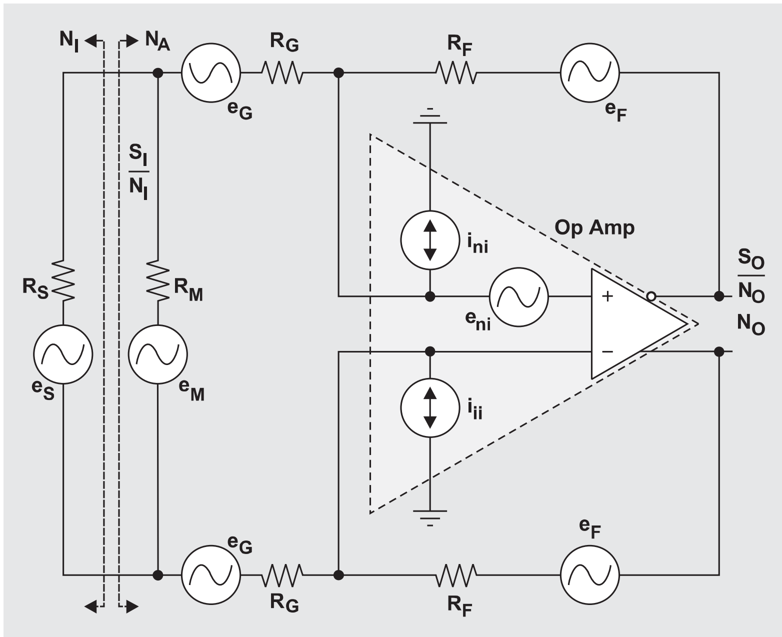


Figure 3. Fully differential noise analysis diagram



*The virtual-short concept simplifies the analysis. Much more work is required to obtain the same results by other means such as nodal analysis.

to the input by their respective scaling factors and summed to find N_A; i.e.,

$$N_A = c_1 e_{ni}^2 + c_2 i_{ni}^2 + c_3 i_{ii}^2 + c_4 e_T^2 + c_5 e_G^2 + c_6 e_F^2, \quad \textbf{(3)}$$

where c_1 through c_6 are the scaling factors.

The op amp's input voltage noise is e_{ni}. It appears directly at the amplifier's input and its scaling factor is 1 or unity, so that $c_1 e_{ni}^2 = e_{ni}^2$.

The op amp's non-inverting input current noise is i_{ni}. It develops a voltage through the parallel combination of R_S and R_T, which appears directly at the amplifier's input, so that

$$c_2 i_{ni}^2 = i_{ni}^2 \left(\frac{R_S R_T}{R_S + R_T} \right)^2.$$

The op amp's inverting input current noise is i_{ii}. It develops a voltage through the parallel combination of R_F and R_G at the op amp's inverting input. By amplifier action, this voltage appears at the amplifier's input, so that

$$c_3 i_{ii}^2 = i_{ii}^2 \left(\frac{R_F R_G}{R_F + R_G} \right)^2.$$

The noise voltage term e_T associated with R_T is equal to $\sqrt{4kTR_T}$. It is divided by the resistors R_S and R_T, so that

$$c_4 e_T^2 = 4kTR_T \left(\frac{R_S}{R_S + R_T} \right)^2.$$

If $R_T = R_S$, then $c_4 e_T^2 = kTR_T$.

The noise voltage term e_G associated with R_G is equal to $\sqrt{4kTR_G}$. This noise is divided by the resistors R_F and R_G and applied to the op amp's inverting input. Again by amplifier action, noise from R_G appears at the amplifier's input, so that

$$c_5 e_G^2 = 4kTR_G \left(\frac{R_F}{R_F + R_G} \right)^2.$$

The noise voltage term e_F associated with R_F is equal to $\sqrt{4kTR_F}$ and appears at the amplifier's output. Dividing by the signal gain gives us

$$c_6 e_F^2 = 4kTR_F \left(\frac{R_G}{R_F + R_G} \right)^2.$$

With all the terms in Equation 3 quantified, we can take the sum to find N_A and use N_A along with N_I in Equation 2 to find F.

Inverting amplifier

Finding the input-referred noise of an inverting op amp amplifier is more cumbersome than finding that of a non-inverting op amp amplifier. The main problem is that the signal gain of the amplifier and the noise gain are different. Figure 2 shows a noise analysis diagram for an inverting op amp amplifier with the noise sources identified.

To find the input-referred noise, it is easiest in some cases to find the output noise and then divide by the signal gain of the amplifier.

The noise voltage delivered to the input from the source is divided by the resistors R_S and R_M in parallel with R_G. Therefore,

$$N_I = 4kTR_S \left[\frac{R_M R_S}{R_S(R_M + R_G) + (R_M R_G)} \right]^2.$$

R_M is typically selected so that $R_M \parallel R_G = R_S$, in which case $N_I = kTR_S$.

The amplifier's input-referred voltage noise is a combination of e_{ni}, i_{ni}, and i_{ii} with associated impedances e_T, e_G, e_F, and e_M. These are all referred to the input by their respective scaling factors and summed to find N_A; i.e.,

$$N_A = c_1 e_{ni}^2 + c_2 i_{ni}^2 + c_3 i_{ii}^2 + c_4 e_T^2 + c_5 e_G^2 + c_6 e_F^2 + c_7 e_M^2, \quad \textbf{(4)}$$

where c_1 through c_7 are the scaling factors.

The op amp's input voltage noise, e_{ni}, at the op amp's non-inverting input appears at the amplifier output as a function of the amplifier noise gain,

$$1 + \frac{R_F}{R_G + \dfrac{R_S R_M}{R_S + R_M}},$$

and is then referred back to the amplifier input as a function of the signal gain, R_F/R_G. Thus,

$$c_1 e_{ni}^2 = e_{ni}^2 \left(\frac{R_G}{R_F} + \frac{R_G}{R_G + \dfrac{R_S R_M}{R_S + R_M}} \right)^2 .^{**}$$

The op amp's non-inverting input current noise is i_{ni}. It develops a voltage through R_T that appears directly at the amplifier's input, so that

$$c_2 i_{ni}^2 = i_{ni}^2 \left(\frac{R_T R_G}{R_F} + \frac{R_T R_G}{R_G + \dfrac{R_S R_M}{R_S + R_M}} \right)^2.$$

It is hard to see how to calculate the op amp's inverting input current noise, i_{ii}. Basically, due to amplifier action, the inverting node is at ground so that no current is drawn through the input resistor R_G. The noise current flows through R_F, producing a voltage at the output equal to $i_{ii}R_F$. Referring to the amplifier's input results in $c_3 i_{ii}^2 = i_{ii}^2 (R_G)^2$.

The noise voltage term e_T associated with R_T is equal to $\sqrt{4kTR_T}$. Just like e_{ni}, it appears at the output as a function

**The gain is actually –RF/RG; but since it is squared, the minus sign is ignored in this analysis.

of the amplifier noise gain and is then referred back to the amplifier input as a function of the signal gain, so that

$$c_4 e_T^2 = kTR_T \left(\frac{R_G}{R_F} + \frac{R_G}{R_G + \frac{R_S R_M}{R_S + R_M}} \right)^2.$$

The noise voltage term e_G associated with R_G is equal to $\sqrt{4kTR_G}$. It is divided by the resistors R_G and R_S in parallel with R_M en route to the amplifier's input, so that

$$c_5 e_G^2 = 4kTR_G \left(\frac{R_G}{R_G + \frac{R_S R_M}{R_S + R_M}} \right)^2.$$

The noise voltage term e_F associated with R_F is equal to $\sqrt{4kTR_F}$ and appears directly at the amplifier's output. Dividing by the signal gain gives us

$$c_6 e_F^2 = 4kTR_F \left(\frac{R_G}{R_F} \right)^2.$$

The noise source e_M associated with the input termination matching resistor R_M is equal to $\sqrt{4kTR_M}$. It is divided by the resistors R_M and R_S in parallel with R_G, so that

$$c_7 e_M^2 = 4kTR_M \left[\frac{R_S R_G}{R_M(R_S + R_G) + R_S R_G} \right]^2.$$

With all the terms in Equation 4 quantified, we can take the sum to find N_A and use N_A along with N_I in Equation 2 to find F.

Fully differential amplifier

Fully differential op amp amplifiers are very similar to inverting op amp amplifiers, and the analysis follows very closely. Figure 3 shows the noise analysis diagram.

The source resistance generates thermal noise equal to $\sqrt{4kTR_S}$. The noise voltage delivered to the input from the source is divided by the resistors R_S and R_M in parallel with $2R_G$. Therefore,

$$N_I = 4kTR_S \left(\frac{\frac{2R_M R_G}{R_M + 2R_G}}{R_S + \frac{2R_M R_G}{R_M + 2R_G}} \right)^2.$$

R_M is typically selected so that $R_M \parallel 2R_G = R_S$, in which case $N_I = kTR_S$.

The amplifier's input-referred voltage noise is a combination of e_{ni}, i_{ni}, and i_{ii} with associated impedances e_G, e_F, and e_M. These are all referred to the input by their respective scaling factors and summed to find N_A; i.e.,

$$N_A = c_1 e_{ni}^2 + c_2 i_{ni}^2 + c_3 i_{ii}^2 + c_4 e_G^2 + c_5 e_F^2 + c_6 e_M^2, \qquad \textbf{(5)}$$

where c_1 through c_6 are the scaling factors.

In this analysis it is assumed that the two input resistors R_G are equal and that the two feedback resistors R_F are equal.

The op amp's input voltage noise, e_{ni}, at the op amp's input appears at the amplifier output as a function of the amplifier noise gain,

$$1 + \frac{R_F}{R_G + \frac{R_S R_M}{2(R_S + R_M)}},$$

and is then referred back to the amplifier input as a function of the signal gain, R_F/R_G. Thus,

$$c_1 e_{ni}^2 = e_{ni}^2 \left[\frac{R_G}{R_F} + \frac{R_G}{R_G + \frac{R_S R_M}{2(R_S + R_M)}} \right]^2.$$

Since the input resistors are equal and the feedback resistors are equal, the op amp's non-inverting input current noise, i_{ni}, and inverting input current noise, i_{ii}, have the same scaling factors. Due to amplifier action, the input nodes of the op amp are ac grounds so that no current is drawn through the input resistors R_G. All the noise current flows through R_F, producing a voltage at the output equal to $i_{ni}R_F$ or $i_{ii}R_F$. Referring to the amplifier's input results in $c_2 i_{ni}^2 = i_{ni}^2 (R_G)^2$ and $c_3 i_{ii}^2 = i_{ii}^2 (R_G)^2$.

The noise voltage term e_G associated with each R_G is equal to $\sqrt{4kTR_G}$. It is divided by the resistors R_G and one-half R_S in parallel with R_M, so that

$$c_4 e_G^2 = 2 \times 4kTR_G \left[\frac{R_G}{R_G + \frac{R_S R_M}{2(R_S + R_M)}} \right]^2.$$

The noise voltage term e_F associated with each R_F is equal to $\sqrt{4kTR_F}$ and appears directly at the amplifier's output. Dividing by the signal gain gives us

$$c_5 e_F^2 = 4kTR_F \left(\frac{R_G}{R_F} \right)^2.$$

The noise source e_M associated with the input termination matching resistor R_M is equal to $\sqrt{4kTR_M}$. It is divided by the resistors R_M and R_S in parallel with $2R_G$, so that

$$c_6 e_M^2 = 4kTR_M \left(\frac{\frac{2R_S R_G}{R_S + 2R_G}}{R_M + \frac{2R_S R_G}{R_S + 2R_G}} \right)^2.$$

As before, with all the terms in Equation 5 quantified, N_A can be calculated and used with N_I in Equation 2 to find the noise factor.

Table 1. Comparison of calculated vs. measured noise figure

OP AMP	CONFIGURATION	e_{ni} (nV)	i_{ni} (pA)	i_{ii} (pA)	R_F (Ω)	R_G (Ω)	R_T (Ω)	R_M (Ω)	CALCULATED NF (dB)	MEASURED NF (dB)
THS3202	Non-inverting	1.65	13.5	20	255	49.9	49.9	—	11.6	11.5
THS3202	Inverting	1.65	13.5	20	255	49.9	—	—	13.6	13.0
THS4501	Fully differential	7	1.7	1.7	392	392	—	56.2	30.1	30.6

Conclusion

The input-referred voltage noise and current noise, along with the circuit configuration and component values, can be used to calculate noise figure. This is a tedious task at best. Setting up a spreadsheet for each topology where component values and op amp specs can be entered is recommended. In this way, various scenarios can be quickly tested. Verification by testing the circuit with a noise figure analyzer is always suggested.

As an example of how well the theory outlined in this article matches test results, the noise figure of three op amp amplifiers configured as previously detailed were measured with an Agilent N8973A noise figure analyzer. Table 1 shows that the results are good, with the input current and voltage noise specifications given as typical values.

Related Web sites

analog.ti.com
www.ti.com/sc/device/THS3202
www.ti.com/sc/device/THS4501

Appendix—Summary of noise terms in op amp amplifiers

Signal input noise (N_I) terms

AMPLIFIER CONFIGURATION	NOISE SOURCE	NOISE CONTRIBUTION
Non-inverting	Source thermal noise	$4kTR_S\left(\frac{R_T}{R_S+R_T}\right)^2$
Inverting	Source thermal noise	$N_I = 4kTR_S\left[\frac{R_MR_S}{R_S(R_M+R_G)+(R_MR_G)}\right]^2$
Fully differential	Source thermal noise	$4kTR_S\left(\frac{\frac{2R_MR_G}{R_M+2R_G}}{R_S+\frac{2R_MR_G}{R_M+2R_G}}\right)^2$

Appendix—Summary of noise terms in op amp amplifiers (Continued)

Device input noise (N_A) terms

AMPLIFIER CONFIGURATION	NOISE SOURCE	NOISE CONTRIBUTION
Non-inverting	Op amp input-referred voltage noise	e_{ni}^2
	Op amp non-inverting input-referred current noise	$i_{ni}^2\left(\frac{R_S R_T}{R_S + R_T}\right)^2$
	Op amp inverting input-referred current noise	$i_{ii}^2\left(\frac{R_F R_G}{R_F + R_G}\right)^2$
	Termination resistor thermal noise voltage	$4kTR_T\left(\frac{R_S}{R_S + R_T}\right)^2$
	Gain resistor thermal noise voltage	$4kTR_G\left(\frac{R_F}{R_F + R_G}\right)^2$
	Feedback resistor thermal noise voltage	$4kTR_F\left(\frac{R_G}{R_F + R_G}\right)^2$
Inverting	Op amp input-referred voltage noise	$e_{ni}^2\left(\frac{R_G}{R_F} + \frac{R_G}{R_G + \frac{R_S R_M}{R_S + R_M}}\right)^2$
	Op amp non-inverting input-referred current noise	$i_{ni}^2\left(\frac{R_T R_G}{R_F} + \frac{R_T R_G}{R_G + \frac{R_S R_M}{R_S + R_M}}\right)^2$
	Op amp inverting input-referred current noise	$i_{ii}^2(R_G)^2$
	Non-inverting bias matching resistor thermal noise voltage	$4kTR_T\left(\frac{R_G}{R_F} + \frac{R_G}{R_G + \frac{R_S R_M}{R_S + R_M}}\right)^2$
	Gain resistor thermal noise voltage	$4kTR_G\left(\frac{R_G}{R_G + \frac{R_S R_M}{R_S + R_M}}\right)^2$
	Feedback resistor thermal noise voltage	$4kTR_F\left(\frac{R_G}{R_F}\right)^2$
	Inverting termination matching resistor thermal noise voltage	$4kTR_M\left[\frac{R_S R_G}{R_M(R_S + R_G) + R_S R_G}\right]^2$

Appendix—Summary of noise terms in op amp amplifiers (Continued)

Device input noise (N_A) terms (Continued)

AMPLIFIER CONFIGURATION	NOISE SOURCE	NOISE CONTRIBUTION
Fully differential	Op amp input-referred voltage noise	$e_{ni}^2\left[\frac{R_G}{R_F}+\frac{R_G}{R_G+\frac{R_S R_M}{2(R_S+R_M)}}\right]^2$
	Op amp non-inverting input-referred current noise	$i_{ni}^2(R_G)^2$
	Op amp inverting input-referred current noise	$i_{ii}^2(R_G)^2$
	Gain resistor thermal noise voltage	$2\times 4kTR_G\left[\frac{R_G}{R_G+\frac{R_S R_M}{2(R_S+R_M)}}\right]^2$
	Feedback resistor thermal noise voltage	$2\times 4kTR_F\left(\frac{R_G}{R_F}\right)^2$
	Termination matching resistor thermal noise voltage	$4kTR_M\left(\frac{\frac{2R_S R_G}{R_S+2R_G}}{R_M+\frac{2R_S R_G}{R_S+2R_G}}\right)^2$